The New Zealand Treasury’s Living Standards Framework - Exploring a Stylised Model

Girol Karacaoglu

New Zealand Treasury Working Paper 15/12

August 2015

DISCLAIMER

The views, opinions, findings, and conclusions or recommendations expressed in this Working Paper are strictly those of the author(s). They do not necessarily reflect the views of the New Zealand Treasury or the New Zealand Government. The New Zealand Treasury and the New Zealand Government take no responsibility for any errors or omissions in, or for the correctness of, the information contained in these working papers. The paper is presented not as policy, but with a view to inform and stimulate wider debate.
ACKNOWLEDGEMENTS

I wish to thank the following individuals for their comments and feedback on the first version of this paper - without in any way implying their endorsement: Jean-Pierre Andre, Joey Au, Geoff Bertram, Hilary Blake, Romina Boarini, Jonathan Boston, Nick Carroll, Kristie Carter, Andrew Coleman, Diane Coyle, Eric Crampton, Paul Dalziel, Shamubeel Eaqub, Brian Easton, Hans-Jurgen Engelbrecht, Lew Evans, Roger Fairclough, Martin Fukac, David Galt, Margaret Galt, Arthur Grimes, Oliver Hartwich, Gary Hawke, Seamus Hogan, Andrew Jackson, Adam Jaffe, Jacek Krawczyk, Robert MacCulloch, Wayne Mills, Jamie Murray, Patrick Nolan, Les Oxley, Weshah Razzak, Chris Ritchie, Paul Rodway, James Roumasset, Michael Ryan, Grant Scobie, Graham Scott, Conal Smith, Robert Stratford, Adolph Stroomberg, Ken Warren, Bryce Wilkinson, John Yeabsley. I also benefited from discussions at the New Zealand Treasury’s Living Standards Group meetings and two workshops, one at the New Zealand Treasury and the other at the New Zealand Productivity Commission.

DEDICATION

I dedicate this paper to the memory of the late, great Sir Paul Callaghan, as a tribute to his vision of New Zealand as a place where talent wants to live. The main aspiration of the paper is to suggest policies (or strategies) that can help us achieve that vision.
Abstract

The New Zealand Treasury’s Living Standards Framework (LSF) is a guide for thinking about good economic, environmental and social policy in an integrated way - policy that aims to enhance individual and communal wellbeing on a sustained basis. This paper presents an evolving stylised model (one possible model) for the LSF; it is work in progress. The model is constructed by weaving together threads from the wellbeing, sustainable development and endogenous economic growth literatures. Its primary aim is to capture all key attributes of the LSF in a unified model. In doing so, I wish to identify the domain of a public policy that aims to enhance collective intergenerational wellbeing, highlight the key complementarities and tradeoffs that we face as a society in this pursuit, and explore the policy options and levers available to the policy makers to relax these tradeoffs and exploit the complementarities to the same end.
Executive Summary

The New Zealand Treasury’s Living Standards Framework (LSF) is a guide for thinking about good economic, environmental and social policy in an integrated way - policy that aims to enhance individual and communal wellbeing on a sustained basis. This paper presents an evolving stylised and unified model for the LSF; it is work in progress. The model is constructed by weaving together threads from the wellbeing, sustainable development and endogenous economic growth literatures.

Using the analysis of Arrow et al (2012, 2013) as a platform, we define the object of interest for public policy as shared and sustained intergenerational wellbeing. The source of wellbeing is “comprehensive consumption”, which includes marketed consumption goods, as well as others such as leisure, arts, health services, and consumption services provided by nature. The ultimate source of comprehensive consumption is “comprehensive wealth”, which comprises stocks of capital assets, broadly defined, that yield income and other sources of wellbeing. These assets include economic capital, human capital, natural capital, and social capital (including the institutions that underpin the way we work and live).

The ultimate purpose of public policy is to help people live better lives, now and into the future. To this end, good policy focuses on ensuring that the wellbeing-generating capacity of capital assets is sustained or enhanced, and shared, which is to say: not eroded by current generations at the expense of future generations (sustainability); is shared in a manner consistent with sustaining or enhancing the capital base (equity); that no particular social group(s) impose their concepts of wellbeing on others, respecting others’ rights to live the kinds of lives they have reason to value (social cohesion); that comprehensive wealth is protected against major systemic risks (resilience); and that the material wellbeing generating potential of capital assets is enhanced (to underpin the economy’s capacity to sustain higher growth).

These are all outcomes with public good (non-rivalry and/or non-excludability) characteristics, and they are the sources of significant positive externalities that public policy can wrap around comprehensive wealth, to enhance our capabilities and opportunities (i.e. substantive freedoms), as individuals and communities, to pursue the kinds of lives we have reason to value - i.e. to expand our collective “wellbeing frontier” [see Sen (2009)]. We have chosen to focus on these particular public goods through reasoned discussion, informed by the broader domains of wellbeing identified by the OECD, the New Zealand Ministry of Social Development (MSD), Statistics New Zealand, and others.
The main instruments through which public policy operates to enhance sustainable and shared wellbeing are targeted at the growth and protection of, and widespread access to, public capital assets, and the equitable distribution of private capital assets in a way that does not blunt incentives for participation in the creation and productive application of wealth. It is through these particular mechanisms that public policy can enhance individuals’ capabilities and opportunities to live the kinds of lives they have reason to value.

Once we specify the generation and protection of shared (across society and generations) and sustainable wellbeing as the main purpose of public policy, and we appreciate the multiplicity and complementarity of spheres of wellbeing, it becomes self-evident that we need to think of economic, social and environmental policies in an integrated way. The LSF motivates integrated policy formulation, and the stylised model is intended to highlight the key complementarities and tradeoffs between key policy outcomes. Ignoring these linkages could lead to policies that harm individual and communal intergenerational wellbeing.

Our stylised model has a single consumable, and internationally tradeable, good that is produced using both "clean" and "dirty" technology. Clean technology works with relatively skilled labour, while dirty technology uses relatively unskilled labour as well as non-renewable natural resources.¹ A very important source of the economic-growth potential of an economy (which underpins its material wellbeing) is productivity growth supported by knowledge-based innovation. New ideas and methods of production generated by scientists and engineers are embodied in human and physical capital assets in the form of productivity improvements and converted, through the production process and market testing, into goods that consumers want. Innovation-embodying human and physical capital can be generated through domestic investment or by importing them.

International human and physical capital are attracted to the small open economy because of its relatively high quality of life (reflecting the quality of its physical environment and social cohesion), and its offer of relatively high material wellbeing. Material wellbeing is positively affected, through both the price premium received for the final good and the real wages earned by skilled labour, by the extent to which production of the single tradeable good uses clean technology.

From an overall wellbeing perspective, a laissez-faire equilibrium (or steady state) is sub-optimal, and so are policy interjections that are solely targeted at increasing economic growth. Material wellbeing is essential for overall wellbeing, and as a ¹ The term “dirty technology” is not used with any derogatory intent, but simply as a convenient means of differentiating between two types of technology with significantly different impacts on the accumulation of human and physical capital, and the preservation of natural capital.
basis for social and political progress, but economic growth cannot be sustained unless it is socially and environmentally sustainable [Duraiappah and Munoz (2012)]. Furthermore, economic progress needs to be broadly based if it is to foster social and political progress [Friedman (2006)].

There is no presumption that the government necessarily knows better or can do better. Our purpose rather is to raise awareness of the wider spheres of wellbeing (including material wellbeing), and highlight their relevance for a policy aimed at enhancing intergenerational wellbeing. In doing so, we also highlight the interdependencies and complementarities between these various spheres of wellbeing.

A particularly promising policy package includes incentivising clean-technology research and penalising the use of dirty technology with a view to switching production towards clean technology - supported by subsidising skilling and education, as well as the immigration of skilled labour, engineers and scientists. This combination of policies would raise both the rate and the quality of sustainable growth by reducing the negative effects of production on the environment and on health. A reduction in inequity would follow as the relative weight of skilled labour and scientists (that are both wealthier and better paid than unskilled labour) increases in the working population. This package would need to be enhanced by poverty-reducing and community-building investments to ensure that social cohesion and resilience to systemic shocks are increased.

Thus the LSF is not anti-growth; it is pro-good growth.
# Contents

Abstract i

Executive Summary ii

1 Introduction 1
   1.1 Purpose of Public Policy ............................................. 1
   1.2 Domain of Public Policy .............................................. 3
   1.3 LSF and Purpose of the Stylised Model ............................. 7

2 A Narrative - Model in Words 9
   2.1 General Setting ..................................................... 9
   2.2 Comprehensive Wealth - Capital Assets .......................... 11
   2.3 Social, Environmental and Economic Externalities ............... 12
   2.4 Actors and Actions ................................................. 15
   2.5 Production .......................................................... 16
   2.6 Evolution and Distribution of Capital Assets .................... 17
   2.7 Ideas and Technology .............................................. 18
   2.8 International Connections ....................................... 19
   2.9 Pulling the Pieces Together .................................... 21
   2.10 Wellbeing-Enhancing Public Policy ............................. 22

3 A Unified and Stylised Formal Model 25
   3.1 Wellbeing, Consumption, Income and Wealth .................... 26
      3.1.1 Generic Model .................................................. 26
      3.1.2 Stylised Model ................................................ 29
   3.2 Production and Wellbeing ......................................... 34
      3.2.1 Generic Model .................................................. 34
      3.2.2 Stylised Model ................................................ 35
   3.3 Technology and Market Structure ................................ 36
   3.4 Distribution and Equity .......................................... 37
      3.4.1 Externalities, Time Horizons and the Government .......... 40
      3.4.2 International Connectedness ................................ 46

4 Model Solution - Equilibrium Properties 48
   4.1 Actors and Actions .................................................. 49
      4.1.1 Utility Maximising Individuals ............................... 49
      4.1.2 Producers of the Final Product ................................ 51
      4.1.3 Machine Producers or Importers .............................. 52
      4.1.4 Scientists, Technology and Productivity Growth .......... 53
   4.2 Market-Clearing Conditions ..................................... 55
      4.2.1 Final Good ....................................................... 55
4.2.2 Labour ........................................... 55
4.2.3 Scientists ........................................ 56
4.2.4 Machines ......................................... 56
4.2.5 Exhaustible Natural Resources ........... 57
4.2.6 International Payments Balance ......... 57
4.3 Equilibrium ........................................ 58
  4.3.1 Laissez-Faire - A Benchmark Model .. 58
  4.3.2 Laissez-Faire - LSF Model ............ 61
4.4 Equilibrium under Wellbeing-Maximisation Policy .......................... 64

5 Concluding Comments and Next Steps .......................... 69

List of Figures

Figure 1 – Living Standards Framework ......................... 7

List of Tables

Table 1 – Summary of Equilibrium Properties of Alternative Models ........ 66
1 Introduction

1.1 Purpose of Public Policy

The ultimate purpose of public policy is to improve people’s lives, now and into the future.

We do not know how each and every individual wishes to live his/her life, nor do we wish to pass judgement on how they should be living their lives. There are an infinite number of possible lives, shaped by personal circumstances, including capabilities, opportunities and preferences, as well as possible cultures, religions, political arrangements, geographical surroundings and so on.

Given this objective and this constraint, and based on previous works summarised in Sen (2009), Stiglitz, Sen and Fitoussi (2009), O’Donnell, et al (2014), Braunerhjelm and Henrekson (2015), Feldman et al (2014) and McCloskey (2014), the New Zealand Treasury’s Living Standards Framework (LSF) specifies the purpose of public policy as enhancing the capabilities and opportunities of individuals to pursue the lives they have reason to value (i.e. to increase their wellbeing), helping them remove the obstacles they face in this pursuit and, in doing so, making sure that we do not blunt the incentives of individuals to do the best they can for themselves [Gleisner et al (2012)].

Although we do not know how individuals want to live, nor do we wish to pass judgement on how they should be living, we cannot ignore the findings of numerous studies, covering a large variety of countries and cultures, about the broader domains of individual wellbeing across many societies. By way of examples, the OECD’s Better Life Initiative [OECD (2013), OECD and Clio-Infra (2014)] focuses

---

2 Needless to say, limits will have to be imposed on those who want to live their lives in total disregard for the wellbeing of current and/or future generations. Such limits must be ethically based, and able to withstand intense and rigorous public scrutiny. My thanks to Jonathan Boston for raising this point.
on the following domains or spheres of individual wellbeing, classified under quality of life (health status, work-life balance, education and skills, social connections, civic engagement and governance, environmental quality, personal security, subjective wellbeing) and material conditions (income and wealth, jobs and earnings, housing). Similarly, the Social Reports produced by the New Zealand Ministry of Social Development (MSD) [NZMSD (various years)] identify ten domains of wellbeing: health, knowledge and skills, paid work, economic standard of living, civil and political rights, cultural identity, leisure and recreation, safety, social connectedness, life satisfaction (earlier versions also included an environment domain). Finally, Statistics New Zealand (2009) identifies three domains in its Framework for Measuring Sustainable Development: environmental responsibility, economic efficiency, social cohesion. In all three cases, the identification of these spheres is evidence-based, in the sense that they are arrived at by extensive consultations with the wider public. They are also informed by the findings of empirical work [see, e.g., Di Tella and MacCulloch (2008)].

One of our two primary purposes is to operationalise the LSF as a guide for good public policy advice. The breadth of the wellbeing domains highlighted above clearly suggests that the sources of human wellbeing are multi-dimensional and complementary in nature. Consequently, the LSF has deliberately adopted a broader, multi-dimensional, and integrated approach to economic, environmental and social policy advice that promotes wider wellbeing on a sustainable basis. It also advocates the use of a multiplicity of complementary policy instruments in the pursuit of this objective.

Our second primary purpose is to formalise the thinking that underpins the LSF by formulating a stylised model that represents it. In doing so, as a first step we follow Arrow et al (2012) and, in the spirit of the LSF, define the object of interest for public policy as intergenerational wellbeing: "When scholars adopt intergenerational wellbeing as the object of interest, their presumption is that at any given date social well-being is not only the well-being of the current generation, but also the potential welfare of the generations that are to follow. The point is to ask whether the society under study is functioning sufficiently well to ensure that some measure of intergenerational wellbeing does not decline" (Ibid, p. 318).

Social wellbeing reflects individual wellbeing. Although there is no entity (i.e. "society") that exists separately from the individuals who live together as a community,
individual wellbeing is affected by how that community functions. Individual wellbeing will be enhanced if people can live the way they want to live; the immediate community and the wider society in which individuals live will have a significant bearing on whether this is achievable.

The spheres or domains of wellbeing identified above also mirror very closely what Arrow et al (2012) refer to as "comprehensive consumption", which includes not only standard marketed consumption goods, but also others such as leisure, arts, health services, and consumption services provided by nature. The ultimate source of comprehensive consumption is "comprehensive wealth" [Arrow et al (2012)]. This refers to the stocks of capital assets, broadly defined, that yield income and other sources of wellbeing now and into the future (ibid, p. 320), to be conceptualised as the discounted present value of our overall future stream of wellbeing. These assets include economic capital, human capital, natural capital, and social capital. "... because the determinants of intergenerational wellbeing are the multitude of capital assets the economy has inherited from the past, the criterion function for sustainable development reduces to a weighted sum of the stocks of those assets - the weights being the marginal contributions of the stocks to intergenerational wellbeing. The weights are therefore the assets' shadow prices, and the weighted sum is the economy's wealth" (ibid, p. 318).

An increase in comprehensive wealth is referred to as "genuine saving" (or "comprehensive investment"), and is represented as a change in each form of capital, valued using shadow prices which reflect the marginal contribution of each capital asset to wellbeing [Hamilton and Hepburn (2014)].

1.2 Domain of Public Policy

If the object of interest is intergenerational individual and communal wellbeing, then the fundamental role of public policy is one of stewardship of comprehensive wealth, for the wider benefit of current and future generations. What would a steward tasked with maintaining or enhancing overall intergenerational wellbeing focus on - what would be the steward's policy domain?

It would want to ensure that the wellbeing-generating capacity of capital assets is sustained or enhanced, and shared, which is to say: not eroded by current generations at the expense of future generations (sustainability); is shared in a manner consistent with sustaining or enhancing the capital base (equity); that no particular social group(s) impose their concepts of wellbeing on others, respecting others’ rights to live the kinds of lives they have reason to value (social cohesion);
that comprehensive wealth is *protected against major systemic risks (resilience)*; and that the *material wellbeing* generating potential of capital assets is enhanced (to underpin *the economy's capacity to sustain higher growth*).

We conceptualise these as outcomes with public good (non-excludability and/or non-rivalry) characteristics, that are the potential sources of significant positive externalities. They are therefore of interest to a public policy that is focused on enhancing wellbeing. These are outcomes that good public policy can wrap around our total comprehensive wealth to enhance our capabilities and opportunities (i.e. substantive freedoms), as individuals and communities, to pursue the kinds of lives we have reason to value [Sen (2009), McCluskey (2014)]. The choice of these particular public goods reflects the broader domains of wellbeing identified by the OECD, MSD, Statistics New Zealand, and others.

It is wellbeing, and not welfare, that is the primary focus of public policy in our framework. The role of a “welfare state” is to deliver welfare – agency is typically assumed to reside with the government. A “wellbeing state” on the other hand aims to expand the opportunities and capabilities of individuals to enhance their own wellbeing by "having a go". The personal agency of individuals as citizens is paramount. Values must be reasoned, and citizens have a responsibility in being actively engaged in pursuing these values; unless their circumstances make it inevitable, they cannot be passive recipients of what they want and value [Dalziel and Saunders (2014), Sen (2009)].

Four questions arise immediately: first, why is there a need for public policy (or deliberate collective action) at all, in enhancing individual and communal wellbeing; second, if there is a role for public policy, what is its domain; third, and closely related to the second, how do we ensure time consistency (ongoing alignment of public policy with collective wellbeing); and finally, what are the instruments available to the government to operate effectively and efficiently in its chosen policy domain?

Why does the government have any stewardship role to play at all viz comprehensive wealth, towards helping enhance wellbeing on a sustained basis? After all individuals, partly through voluntary cooperation and exchanges with others, can and do invest in their own economic and human capital, and make all sorts of arrangements to manage associated risks (partly through the purchase of insurance

---

5 An equivalent classification, using a different language, distinguishes between the mid-20th century “entitlement state” and the early-21st century "enablement and self-empowerment state” [Cadogan (2013)].

6 Note that, *the government* is the generic term we use to refer to any collective agency through which we may wish to organise the delivery of certain products or services; it does not necessarily refer to the central government.
contracts), to sustain their own and their families’ wellbeing into the future. They also voluntarily form communities and associations to create the collective goods and services they value.

A possible answer relates to the fact that the types of public goods referred to above, that are potential sources of significant positive externalities, may be under-provided. Because the returns from investing in the capital assets and institutions that generate these spheres of wellbeing (“social goods”) will not be fully captured privately, this may lead to under-investment in, and over-use and/or under-protection of, the components of comprehensive wealth in the absence of deliberate collective action [Miller (2006)]. There is no suggestion that the benefits associated with these positive externalities will not be provided at all; however, they may be under-provided. Through appropriate and deliberate collective action, we may be able to make markets, communities, and institutions work more effectively and efficiently in delivering these public goods. This is a possibility, and not a certainty.

This brings us to the issue of “time consistency”. “Time consistent policies are not policies that are never changed, but policies where any changes required by new circumstances are consistent with maintaining the original purposes of the policy. They are important for socially desirable performance of the private and public sectors. This is because they provide stability that enables individuals and the state to plan for the future. We argue that time consistency is achievable if intergenerational arrangements between the state and the populace are treated as relational contracts. A relational contract is quite different from a legal contract since the latter typically impose specific constraints on arrangements. A relational contract does not define specific constraints but rather a process for developing and changing rules by which all parties agree to abide.” [Evans and Quigley (2013), p.ii]

The domain of government policy in our setting is to help us collectively extract maximum value (in a wider wellbeing sense) from our overall comprehensive wealth (or stocks of capital) by helping shape, grow, share and protect these capital assets for the benefit of current and future generations. In other words, its target is the effective and efficient accumulation and management of comprehensive wealth towards expanding our overall “wellbeing frontier”. In doing so, and to ensure time consistency, public policy needs to be informed by and aligned with evolving public preferences across complementary spheres of wellbeing.

This is why democratic institutions are so critical in helping us resolve tensions through public reasoning and deliberation [Bertram (2013), Sen (2009), Walzer
It is the democratic process, supported by appropriate institutions, that provides the forces that push towards an alignment of collective action (implemented through the government as our agent) with evolving private and communal interests (i.e. government action is endogenous). This ensures time consistency.\(^7\)

What instruments are available to a "wellbeing state" to pursue its purpose? Given the multiplicity and complementarity of the spheres of wellbeing, a variety of complementary policy instruments are required. These instruments range from direct investments by our collective agent (the government), alone or in partnership with private individuals, communities or businesses, to a series of interventions (e.g. through regulation, taxation, subsidies, fees) aimed at influencing private and community behaviour in a direction that is better aligned with wider wellbeing. Certainly in the New Zealand context, the set of instruments includes the size and structure (or composition) of the Crown balance sheet. These instruments all operate through their influence on the "equations of motion" of both the capital assets that comprise comprehensive wealth, and of the public goods that are wrapped around these capital assets.

At the foundation of all this is: universal access to basic income, education, health and housing, the provision of which is the policy domain of a "welfare state", complemented by a set of institutions that ensure that the incentives for all to do the best they can for themselves, their families, their communities and their shareholders remain unblunted; well defined and protected individual rights; and a set of institutions that support a well-functioning society and economy.

Thus, in our stylised model, the collective agent who delivers public policy has both "welfare state" and "wellbeing state" attributes – and these are complementary. Whatever the domain of public policy is, efficient and effective delivery of public services is essential for sustainable wellbeing - thus support for wider and sustainable wellbeing is not a vote against efficiency.\(^8\)

---

\(^7\) This is my interpretation of "time consistency" in the context of this paper; there is no suggestion that Evans and Quigley would agree with this interpretation.

\(^8\) The proper domain for public policy remains an area of intense debate. For an argument in favour of a narrower scope for public policy, focused on "efficiency", see Buchanan and Hartley (2000). For an argument favouring a possibly broader or different shaped scope for public policy, including interventions such as "nudgeing" etc reflecting behavioural insights, see O’Donnell et al (2014). As Spence (2014) puts it, "If countering inequality and promoting intergenerational opportunity introduces some marginal inefficiencies and blunts some incentives, it is more than worth the price. Public provision of critical basic services like education or health care may never be as efficient as private-sector alternatives; but where efficiency entails exclusion and inequality of opportunity, public provision is not a mistake" [Spence (2014)]. See also Helm (2010).
1.3 LSF and Purpose of the Stylised Model

Figure 1 is an attempt to capture the essence of the LSF. In the middle of the diagram, we have the capital assets that comprise the key components of comprehensive wealth. Wrapped around these capital assets are five key social outcomes or public goods that public policy aims to provide or increase, to help us extract most value (wellbeing) from comprehensive wealth. Both the choice of the key components of comprehensive wealth, as well as the public goods that are significant sources of positive externalities that envelop these capital assets, have been selected through extensive reasoned discussion, that has in turn been informed by the spheres of wellbeing that the MSD, Statistics New Zealand, the OECD, and others, have identified. The glue that binds comprehensive wealth to these public goods is stewardship - the stewardship of our capital assets for the benefit of current and future generations.

Our aspiration is to demonstrate the existence of a coherent and unified stylised model (one possible such model), which captures all key attributes of the LSF. In doing so, we also wish to integrate the "subjective wellbeing", "capabilities" and "opportunities" approaches to wellbeing in one model [Baujard and Gilardone (2015), Ferreira and Peragine (2015), O’Donnell et al (2014), Robeyns (2005)].

The primary motivation for doing so is to highlight the most important collective or social outcomes (public goods) that a policy-adviser should be focusing on, in shaping and managing our comprehensive wealth towards enhancing intergenera-
tional wellbeing. These are the five dimensions that surround the capital stocks in the diagram. We also want to highlight and study the interdependencies (substitutabilities and/or complementarities) between these major outcomes, as well as the policy-instruments used to deliver them, as we pursue the enhancement of intergenerational wellbeing. Having identified these, public policy would then aim to minimise the tradeoffs, and/or maximise the complementarities, involved towards enhancing aggregate wellbeing through the use of appropriate policy levers.\(^9\)

I make no claims to originality. The evolving stylised model I present weaves together threads from the existing literature on wellbeing, sustainable development and endogenous economic growth. All the ingredients of the model are already out there, in works usefully summarised, and expanded upon, by Acemoglu, et al (2012), Acemoglu, et al (2014), Arrow et al (2012, 2013), Chichilnisky (1997), Jones and Vollrath (2013), Krugman (1979), and Turnovsky and Mitra (2013). I borrow heavily from all these works; in fact, in a lot of places, I copy directly from them. My only aspiration, and possible contribution, is to integrate the key insights and methods presented in these works into a coherent and unified model that underpins the New Zealand Treasury’s LSF.\(^10\)

The main purpose of a stylised model is not to capture all the relevant detail that may apply to particular circumstances, but rather to provide a structure for thinking, in a rigorous way, about the matter at hand - which, in this case, is the role of public policy in enhancing intergenerational individual and communal wellbeing, and the channels through which it operates to serve this purpose. We use the domain of public policy as a prism for determining the level of detail that is included in the model. By way of example, since influencing individual preferences between private marketable consumption goods is out of scope, no such detail is included in the model except with reference to the choice between private consumption goods, leisure and health services. On the other hand, since the allocation of human and physical capital across individuals is critical for potential wellbeing, individuals in our stylised model are heterogeneous with respect to their ownership of these forms of capital. I welcome criticism that highlights any dimensions that have been excluded from the model, but that are deemed to be essential for the policy matters under consideration.\(^11\)

---

\(^9\) See also Galor and Weil (1999) on the desirability of building unified models.

\(^10\) An alternative model can be based on viability theory [Krawczyk and Judd (2015), Krawczyk and Kim (2014)]. Yet another potential approach to the problem at hand is provided by overlapping generation models [De La Croix, David and Michel, Philippe (2002)].

\(^11\) One such potential criticism is the absence of stochastic, or probabilistic, uncertainty (or risk) in the model. I return to this point later in the paper in discussing risk management as resilience-enhancement.
The model is based on the following fundamental ideas: the objective and focus of human activity and of public policy is intergenerational and shared wellbeing; the source of wellbeing is comprehensive consumption that is itself sourced from comprehensive wealth; the distribution of capital assets is the main driver of the distribution of capabilities and opportunities for the pursuit of wellbeing (hence equity); this distribution is significantly influenced by access to public capital and the ownership of private capital; the evolution of private physical capital reflects private saving, and that of human capital reflects investment in health, education and skilling; and public policy ultimately influences intergenerational and shared wellbeing by influencing the growth, allocation and preservation of these capital assets.

The rest of the paper is structured as follows. In the next section, I provide a narrative that captures the essential features of the more formal model to be presented in subsequent sections. Section three provides a mathematical formulation and section four some of the equilibrium properties of the model. Section five offers concluding comments and suggests next steps. To re-emphasise, this is work in progress - it is exploratory; I welcome feedback.

2 A Narrative - Model in Words

2.1 General Setting

The LSF does not make any sense at all outside of a social context. It is a framework for thinking about wellbeing-enhancing social, environmental and economic policies in an integrated way, and the distinctive role of government in this context. The main focus of such policies is to increase intergenerational individual and communal wellbeing on a sustainable basis, by enhancing the capabilities and opportunities of individuals to pursue the kinds of lives they have reason to value.

In our model, the government is not “a machine that computes optimal solutions to social welfare maximands” [Romer (1988), p. 167], nor a social engineer or despot that pursues an “externally defined, supra-individualistic ideal” [Buchanan (1986), p. 5] - concerns that modern public choice theorists express most strongly. Government policies can fail, just like markets or communities may fail. Governments are agents through whose assistance and support collective public goods are provided; these public goods may help enhance (note, enhance or improve - not equalise) the opportunities and capabilities of individuals to improve their private
wellbeings. Such assistance or support will often take the form of interventions that are intended to make markets and communities work more effectively and efficiently in delivering valued public goods.

There are three clusters of actors or agents in our stylised model: individuals, firms, and a government through whom individuals implement collective actions. These agents live and work in a small open economy that is connected to the rest of the world through trade, as well as flows of physical capital, people and ideas.

At the core of the model is the interaction between wellbeing, consumption, wealth, production, and public policy. Wellbeing is a function of comprehensive consumption, which includes not only standard marketed consumption goods, but also (among others) leisure, arts, health services, and consumption services provided by nature. Intergenerational wellbeing is the discounted present value of the utilities derived by current and future generations from comprehensive consumption [Arrow et al (2012), Hamilton and Hepburn (2014)].

The source of comprehensive consumption is comprehensive wealth - stocks of capital assets. Some capital stocks influence intergenerational wellbeing both indirectly, through the consumable goods and services they help produce, and also as direct sources of utility. Components of natural capital (such as a clean environment) and human capital (such as good health) are just two examples.

In addition, in a social and intergenerational setting, individuals receive positive benefits from public goods with potentially significant positive externalities, such as social cohesion, equity (across society and generations), sustainability of sources of wellbeing, resilience to major systemic risks, as well as increases in the growth potential of the economy. The choice of the particular public goods and associated externalities we focus on (i.e. the domain of public policy) is informed by the preferences of the public at large, as revealed or discovered by various mechanisms (such as surveys) - and the evolution of these preferences is aligned with public policy through the democratic process. Because the sources of all these externalities have “public good” attributes, they may be under-provided, and/or their sources may be under-protected, by private individuals, businesses and communities if left to their own devices.

Micro-foundations of the model (i.e. the specification of what motivates individual actors and communities) is important because some policy interjections are targeted at incentivising private individuals, communities and firms to adjust their behaviours so that they are more aligned with the public good. Others involve direct investments by the government (on its own as our collective agent, or in
partnership with private agents and community institutions), in building, shaping, managing, distributing, and / or protecting our capital assets.

2.2 Comprehensive Wealth - Capital Assets

In the stylised model, there are four broad categories of capital assets, collectively referred to as comprehensive wealth, which are direct and/or indirect sources of wellbeing: economic capital, human capital (broadly defined as we will see shortly), social capital, and natural capital.\(^\text{12}\)

*Economic capital* encompasses produced (human-made) machines, buildings etc directly used in production, as well as economic infrastructure. Economic infrastructure comprises physical infrastructure (roads, bridges etc), which has public good attributes, providing and supporting transport, communication, payments, energy and other networks and services shared by multiple users, underpinning a variety of production and distribution activities.

The World Economic Forum’s *Human Capital Report* (2013) defines a nation’s *human capital* endowment as, “the skills and capacities that reside in people and that are put to productive use” (p.3). It then proceeds to construct an index for human capital that is based on four “pillars”: education, health and wellness, workforce and employment, and enabling environment (p.4). The stock of knowledge embodied in human beings is also included in human capital. Directly or indirectly all these pillars are captured in the formal model presented in Section 4 below.

Thompson (2015), after providing a brief survey of the use of the term *social capital* in business and economics contexts, defines it as, “the set of network-based processes, built upon generalised trust, that influence the ability of a country’s inhabitants to share, cooperate and coordinate actions. In short, social capital is generalised trust and its networks” (p.3). Thus, social capital can be embodied in institutions (including legal, regulatory and financial institutions, and governance) that support social harmony, economic and social opportunity (i.e. social infrastructure) and/or in individual expectations of tolerance, trust and mutual respect.\(^\text{14}\)

As such, it contributes to both the potential growth rate of the economy (through

\(^{12}\) Stiglitz (2015) makes a very compelling case for carefully distinguishing between capital (in the sense of produced assets such as machines) and wealth (including “land or other ownership claims giving rise to rents”). Our broader definition of both wealth and capital assets should hopefully address his concerns.

\(^{13}\) Others, such as Gleeson-White (2014) and Sachs (2014) refer to six categories of capital but, depending on how we define the various categories of capital, we are essentially referring to the same types of capital; there are no substantial differences here.

\(^{14}\) See also Hamilton and Hepburn (2014); Jones and Vollrath (2013), chapter 7.
innovation and productivity growth) and to social cohesion [Algan and Cahuc (2013), and Varvarigos and Xin (2015)].

Natural capital (comprising both non-renewable and renewable resources), includes underground assets (minerals, fossil fuels), commercial land, fish stocks, natural land, and ecosystem services that all these resources provide [Hamilton and Hepburn (2014)].

Public policy can contribute to sustainable and shared intergenerational wellbeing by influencing the growth, allocation and preservation of these capital assets, which comprise overall comprehensive wealth.

### 2.3 Social, Environmental and Economic Externalities

The dimensions of the LSF, seen from a social, environmental and economic policy perspective, are public goods (potential sources of significant positive externalities) wrapped around the capital assets that are the sources of wellbeing. They are the main social outcomes that public policy is pursuing, through its influence on the growth, allocation and preservation of components of comprehensive wealth, towards enhancing sustainable and shared intergenerational wellbeing.

Resilience, in the context of the LSF, refers to the capability of the economy and society to respond to, and continue to operate reasonably effectively and efficiently in the aftermath of, significant systemic shocks to comprehensive wealth. It is achieved through risk management policies that reduce the impact of the major (systemic) shocks that we are collectively most likely to be exposed to. We first identify such systemic risks, and then decide how much we are willing to collectively invest towards building resilience to them [ADB (2013)]. To use and adapt the language of Hansen (2014), here we are focused on "uncertainty outside models" but stop short of assigning probabilities to the occurrence of the types of systemic shocks we are most concerned about - because there is no way of knowing these probabilities.

A system may show resilience to major systemic risks not necessarily by returning exactly to its previous state following a shock, but instead by finding different ways to carry out essential functions; that is, by adapting. We therefore think of a resilient economic and social system as one that has the capability to: withstand sudden shocks, adapt to changing contexts, and recover to a desired state (either
the previous one or a new one), while preserving the continuity of its operations. Thus resilience encompasses both recoverability (the capacity for speedy recovery after a crisis) and adaptability (timely adaptation in response to a changing environment).

We classify resilience-enhancing measures against systemic shocks into three clusters - resilience against systemic economic, natural and social risks. By way of examples, investments (including regulations) towards enhancing financial and macroeconomic stability increase resilience to potential economic risks; regulations and investments targeted at reducing the impact of earthquakes, the sources of climate warming, and the likelihood of imports of diseases from the rest of the world, increase resilience to systemic natural risks; and ensuring universal access to basic income, health services, education and housing increases the resilience of our social infrastructure.

We note later that the presence of such resilience-increasing measures against systemic risks allows individuals to be prepared to take more risks, thus potentially enhancing the economic-growth potential of the economy in aggregate - and possibly the wider wellbeing potential of our collective comprehensive wealth.\(^{15}\)

Social cohesion refers to our ability and willingness to live and work together in social harmony, comprising tolerance, trust and mutual respect. It reflects a state of social being where our differences in all its dimensions are respected, embraced and celebrated. We can enhance social cohesion, as well as the growth potential of the economy and overall resilience, by deliberately investing in social infrastructure (or social capital).

The promotion of social and intergenerational equity in our framework is not a pursuit of equality of outcomes or of opportunities. The pursuit of equality as such, as a policy aim, would be fruitless [Buchanan and Hartley (2000), Walzer (1983)]. McCloskey (2014) distinguishes between the French Enlightenment focus on the pursuit of equality of material outcomes, versus the Scottish Enlightenment version which aspires to give everyone the opportunity “to have a go” at living the kinds of lives they have reason to value. As Atkinson (2015) emphasises, in an intergenerational context, this distinction may not be that useful since today’s outcomes determine tomorrow’s opportunities: “Inequality of outcome among today’s generation is the source of the unfair advantage received by the next generation. If we are concerned about equality of opportunity tomorrow, we need to be concerned about inequality of outcome today” (p.12).

\(^{15}\) I want to specifically thank my colleague Ken Warren at the New Zealand Treasury for his insights on risk management as resilience enhancement, particularly in the context of dealing with systemic risks from a policy perspective.
It is the distribution of opportunities and capabilities among individuals and communities, in a way that gives them a fair chance of pursuing the kinds of lives they have reason to value, today and into the future, that is at the core of our concept of equity. The allocation of private wealth (in the form of the ownership of physical capital, human capital, and some forms of natural capital such as land), the evolution of the production technology used in producing the final output, and public policy that enhances and provides widespread access to public wealth, all have a major bearing on equity defined in this way. In our stylised model, increasing equity has a direct positive benefit on wellbeing through enhanced social cohesion. In addition, policies that incentivise skilling, innovation-supporting education and R&D, and the switch of production technology towards "clean technology", all have positive benefits for both equity, and the quality and sustainability of potential economic growth.16

There is a critical difference between the pursuit of equity and the pursuit of poverty-reduction. The first (equity) is a relative concept centred on opportunities and capabilities; the second (poverty) is an absolute concept centred on deprivation – not only income deprivation per se but (multi-dimensional) wellbeing deprivation [Farina (2015)]. They have separate, and distinctive, influences on social cohesion in our stylised model. Complementary policies are required to reduce poverty while promoting equity. Both concepts (equity and poverty) are ultimately about wellbeing and expressed in terms of wellbeing in the formal model that is presented in the next section.

The concept of sustainability used in this paper is sourced from the literature on "sustainable development”. It is defined in terms of our capacity to bequeath capital assets (comprehensive wealth) to future generations so that they can generate a level of wellbeing (in its broadest sense) that is at least as high as our own. Following Arrow et al (2012), "we take intergenerational wellbeing to be the object of interest in sustainability analysis" (p. 319). In operationalising this, Arrow et al note that since the source of potential wellbeing is comprehensive wealth, sustainability is achieved at any point in time if comprehensive wealth at constant shadow prices is non-decreasing at that time. Ultimately, however, sustainability (just like equity and poverty) is defined in terms of wellbeing - maintaining or enhancing overall wellbeing across time.

Positive economic externalities refer to all the positive benefits that flow from economic and social infrastructure, as we have broadly defined them above,

16 In a similar vein, Acemoglu and Robinson (2014) emphasise that, "... inequality should not be thought of as always summarized by a single index, such as the Gini index or the top 1% share. Rather, the economic and political factors stressed here determine the distribution of resources more generally ..." (p. 16)
towards increasing the growth potential of the economy. This is achieved by enhancing the completeness, size, and effective and efficient operation of markets, as well as the stability of the economic and financial environment, and thereby supporting the allocation of resources to productive activities [Jones and Vollrath (2013), chapter 7].

The formal model presented in the next section captures the strong interdependencies between the public goods that are the sources of significant economic, social and environmental externalities. By way of example, both higher resilience to systemic (economic, social and environmental) risks and stronger social cohesion have positive effects on sustainability. Similarly, increased resilience has a positive effect on the growth potential of the economy. And policies that are able to increase (cross-society and/or intergenerational) equity and reduce poverty, separately and distinctly, contribute to both resilience and social cohesion.

2.4 Actors and Actions

In the stylised model, individuals are intergenerational wellbeing maximisers. Firms are profit maximisers. Government is a generic term we use to refer to institutions that have been deliberately created to coordinate and give effect to activities we decide to pursue collectively rather than individually, with a view to enhancing our individual and communal wellbeing.

Since individual wellbeing partly depends on the consumption of private (marketed) consumption goods, individuals need to generate market income to be able to purchase and consume these goods. They can do so as unskilled or skilled labourers, or as scientists/researchers/engineers, or as entrepreneurs and business people in our model.

Income can also be generated from the ownership (as distinct from the production) of private capital, in the form of physical capital (i.e. the machines used to produce the final good), financial capital, human capital and some forms of natural capital (such as land). In the stylised model we introduce in the next section, individuals differ in (i.e. are heterogeneous with respect to) their endowments of human and physical capital; for simplicity, we do not have private financial capital or land in the model. Using Amartya Sen’s terminology, in our stylised model individuals’ opportunities and capabilities to generate higher and sustainable wellbeing for themselves and for future generations are positively affected by their ownership of various forms of private capital, as well as their access to the services of different forms of public capital.
The distinctive role of government is to coordinate and/or undertake activities that individuals have decided to pursue collectively; thus government is by definition the agent for collective action. A primary (but by no means the exclusive) purpose of these activities is to generate positive externalities, and eliminate negative externalities, associated with public goods, partly through the establishment of appropriate institutions (including private property rights, markets, regulations, taxes and fees) that incentivise private agents to internalise such externalities. In addition, the government can undertake collectively-funded investments (on its own or through private-public partnerships) towards generating positive externalities or eliminating negative ones. The government can also use scale (e.g. the size of its balance sheet) to generate positive externalities through its contribution to intergenerational equity (e.g. through retirement income policy). Finally, although this is more contentious, the government may be assigned the task of deliberately pursuing actions, taking a longer-term view than private individuals may choose to do, that will promote the wellbeing of future generations yet to be born, however far into the future that may be. All such government (i.e. collective) activities are intended to enhance wellbeing either directly (say through improved social cohesion) or indirectly by increasing the production of material goods, and they are funded by taxing the income generated from the production of the material goods.

2.5 Production

Given our lack of interest in influencing the diversity of individual preferences across private marketed consumer goods, we assume that there is a single final produced output that is either consumed, or exported, or saved (i.e. invested = converted into the machines used to manufacture the final output). Such conversion can take the form of domestic manufacture, or imports, of the required machinery. Firms use physical capital (human-made machines), unskilled labour, human capital (in the form of skilled labour), and natural capital (in the form of exhaustible natural resources) to produce this output. All economic activity (including costs of economic activity) is measured in units of this single output.

Two types of technology are used in the production of the (single) final output: (relatively) "clean" (new) technology and (relatively) "dirty" (old) technology. This language ("clean" vs "dirty" technology) is used for convenience only, to differentiate between technologies that are more or less skill- and natural-resource
intensive. These two technologies are assumed to be "gross substitutes" in the production of the final output.

The machines (physical capital) used in the production of the final output can be imported, or manufactured at home. In both cases, they are "funded" through saving (the portion of the final output that is not consumed or exported). When these machines are manufactured at home, the transformation of the output that is saved into machines occurs within a single period of time, through a very simple production process (another instance of gross simplification, to be able to focus on our core purpose).

Private inputs used in production are deemed to be "gross substitutes"; and so are the two categories of private goods used in consumption (marketed consumer goods and leisure), conditional on "good health". On the other hand, the sources of positive externalities (i.e. public or social goods) are assumed to be "gross complements".

From a wider wellbeing perspective, among other things, we are particularly interested in the evolution of "public capital" (i.e. social infrastructure, natural capital, and economic infrastructure), and the distribution of "private capital" (i.e. human capital and physical capital).

### 2.6 Evolution and Distribution of Capital Assets

The objective and focus of human activity and of public policy is intergenerational and shared wellbeing.

In the stylised model, the source of wellbeing is comprehensive consumption, that is itself sourced from comprehensive wealth. The distribution of capital assets is the main driver of the distribution of capabilities and opportunities for the pursuit of wellbeing. This distribution is significantly influenced by access to public capital and the ownership of private capital.

The "equations of motion" that describe the evolution of capital assets all have the same structure - terms that identify the sources of the growth and depreciation of each asset over time, and terms that allow for their regeneration through appropriate investments (where regeneration is possible). Public policy, to the extent that this is required and justified, ultimately influences intergenerational and
shared wellbeing through its impact on the growth, allocation and preservation of public and private capital assets, operating through the parameters of these equations of motion, as well as of those that define the evolution of the public goods that are wrapped around these capital assets.

2.7 Ideas and Technology

The ultimate source of progress in our individual and communal wellbeing is the generation or importation, as well as liberation, of ideas and knowledge, supported by institutions and ethical values that allow and encourage their conversion into goods and services that individuals value. The very significant progress achieved over the last few centuries has been based on the widening adoption of two ideas, the new economic idea of liberty for ordinary people and the new social idea of dignity for them (a dignity that is not based on social status) [McCloskey (2014), p.7].

In the stylised model, economic growth is a source of improvements in general material wellbeing. A very important source of the growth potential of an economy is knowledge-based innovation - knowledge generated at home or abroad. This knowledge is embodied in human and physical capital assets in the form of productivity improvements and converted, through the production process and market testing, into goods that consumers want.

The growth in the number of (domestically grown or immigrant) scientists and engineers, their success rate in innovating, and the productivity impact of that innovation are the key drivers of economic growth in our model. The research sector, which is comprised of scientists and engineers, generates new ideas and knowledge. Some of these, if adopted because they are commercially lucrative, lead to productivity improvements in machines using the "dirty" technology, whereas others lead to further improvements in the "clean" technology. Entrepreneurs and engineers work collaboratively to give effect to these improvements, complemented and supported by economic infrastructure provided or coordinated through collective action (i.e. the government). This is one channel through which new ideas and knowledge may lead to productivity improvements.

18 “The modern world was made by a revolution in ethical judgments about commercial virtues and vices, in particular by an up-valuation of market-tested betterment - ... the enrichment of ordinary people, by ordinary people, for ordinary people.” [McCloskey (2014), pp. 5-6]

19 See Bitzer et al (2014), Bosetti et al (2015), Czaika and Parsons (2015) and D’Albis et al (2015) for recent work on the links between immigration, innovation, and economic growth; these works also have very useful summaries of related previous literature.
Other ideas and knowledge lead to productivity improvements through enhancements in human capital. Scientists and engineers are part of our human capital. A country’s scientific base (and stock of ideas and knowledge) can be enhanced through investment in human capital or augmented by inward migration. Thus ideas and knowledge can be generated within the country or imported. The vehicles for importing these can be in the form of human capital (e.g. scientists) or physical capital (e.g. machines).

Ideas and knowledge need not take the form of inventions; they may also take the form of smart applications of existing or new technology, developed domestically or imported from overseas, combined and adapted to suit domestic circumstances [Bruton (1998), Porter and Heppelman (2014)].

Ideas and knowledge are necessary but not sufficient for promoting economic growth, and overall wellbeing on a sustained and equitable basis. What is required is an institutional framework that incentivises their generation as well as their connection to entrepreneurial effort. The consequent “creative destruction”, fuelled by competition, is what provides the dynamics of some of the endogenous growth models; it is especially effective in promoting growth in economies operating with frontier technologies [Aghion et al (2014)].

Three complementary pillars are required: the accumulation, investment and upgrading of ideas and knowledge; complemented by implementation mechanisms and incentive structures that enable and encourage knowledge to be exploited such that growth and societal prosperity are achieved; in turn enabled and supported by the relevant economic infrastructure [Braunerhjelm and Henrekson (2015)].

Skilled labour is required to operate the machines that embody the new technology, and through upskilling individual labourers can increase their material rewards.

## 2.8 International Connections

This small open economy is connected to the rest of the world through trade in goods and services, as well as flows of physical capital, people and ideas. As stated above, the vehicles for importing new ideas and knowledge from overseas can come in the form of human capital (e.g. scientists or engineers) or physical capital (e.g. machines).

The final good (the single representative of “goods and services” in the stylised model) can be exported, consumed at home, or saved (= invested). Physical capital
(i.e. machines used in the production of the final output), can be manufactured at home or imported. People can emigrate or immigrate. Immigrants can be unskilled labourers, skilled labourers, engineers or scientists.

Imports of machinery are associated with local or overseas entrepreneurs bidding for ideas generated in the small open economy or abroad and, if successful, obtaining the monopoly rights for producing or importing the machines embedding the new ideas. To the extent that the small open economy operates at the frontier of "clean" technology, it is able to exercise influence on the prices of the machines it imports.

People emigrate to the small open economy from their own countries because they are attracted by "the quality of life" there, and/or they obtain higher economic rewards (real wages or return to research) by doing so. In the stylised model we present below, "quality of life" is captured by social cohesion and the quality of the natural environment.\textsuperscript{20}

The single, internationally tradeable and consumable, final good can be produced with different mixes of "clean" and "dirty" technology. The greater the weight of "clean" technology in producing the final good, the higher the price-premium (quasi-rent) the producers earn from the sale of the good on both the domestic and international markets. This in turn has a positive influence on both the real income of skilled labour used to operate the "clean" technology, and the return earned on the machines (physical capital) that embody the "clean" technology. This positive differential also provides an incentive for international skilled labour, scientists, engineers, as well as "clean" machines, to move to the small economy [Buera \textit{et al} (2015); Casey and Galor (2014)].

Accommodating the inflow of skilled labour, scientists and engineers from other countries in the stylised model, also helps highlight the (hopefully temporary) tensions between economic growth and social cohesion. The inward migration of skilled labour, or scientists and engineers, while welcome by business because it assists with innovation and productivity growth, at the same time potentially creates social tensions, and puts pressure on social and economic infrastructure [Collier (2013)].

\textsuperscript{20} See Ravlik (2014) for evidence supporting the importance of both economic opportunities and quality of life (broadly defined) in the choice of destination countries for migrants.
2.9 Pulling the Pieces Together

In their capacity as economic agents (as consumers, producers, employees, entrepreneurs, engineers and scientists), human beings interact with each other through various types of markets, with a view to improving their material wellbeing.

Spheres of wellbeing extend well beyond the material dimension. Wellbeing-maximising individuals take this into account in different ways in making their decisions - e.g. in allocating their time between leisure and income-generating work. Individuals, again in pursuit of higher wellbeing, deliberately interact with others in all types of non-market social settings as well (e.g. churches, schools, charitable and civic institutions).

In addition, there are spheres of individual wellbeing that can be most effectively and efficiently enhanced through deliberate collective action by an appointed or elected collective agent (i.e. the government). The domain of this activity will be different for different societies, and will vary over time [Cadogan (2013), Tanzi (2011)].

In this paper, we construct a stylised model that deliberately accommodates different (complementary) spheres of wellbeing, and in that setting explores the role of public policy in enhancing individual and communal wellbeing on an equitable and sustainable basis.

The model is driven by intertemporal wellbeing-maximising individuals, whose wellbeing depends on comprehensive consumption (sourced from comprehensive wealth) and who, to that end, interact with each other as economic agents through markets that clear, but also deliberately (through individual or collective action) strive to augment sources of non-material spheres of wellbeing as well.

The model solution provides a path, over time, for outcomes of economic and non-economic dimensions of wellbeing, as well as overall wellbeing.

We can compare the wellbeing outcomes across model solutions that do and do not incorporate deliberate collective actions to manage various externalities.

From an overall wellbeing perspective, a laissez-faire equilibrium (or steady state) is socially sub-optimal, and so are policy interjections that are solely targetted at increasing economic growth. Material wellbeing is essential for overall wellbeing, and as a basis for social and political progress, but economic growth cannot be sustained unless it is socially and environmentally sustainable [Duraiappah and
Munoz (2012)]. Furthermore, economic progress needs to be broadly based if it is to foster social and political progress [Friedman (2006)].

There is no presumption that the government necessarily knows better or can do better. Our purpose rather is to raise awareness of the wider spheres of wellbeing (including material wellbeing), and highlight their relevance and complementarity for a policy aimed at enhancing intergenerational wellbeing. Policy interjections, where relevant and appropriate, that ignore these interdependencies will lead to sub-optimal outcomes.

### 2.10 Wellbeing-Enhancing Public Policy

When the object of interest is material wellbeing, delivered through economic growth, it might at first be intuitive to think of the other dimensions highlighted by the Living Standards Framework as constraints on the growth potential of an economy. If, instead, as we do in this paper, one defines the object of interest as the enhancement of intergenerational wellbeing, then these dimensions will naturally be seen as complements towards increasing our individual and communal wellbeing on a sustainable basis - they are all essential, complementary, ingredients of wellbeing [van den Bergh (2015)].

In this context, the LSF, as a guide to good policy, can serve at least three separate but possibly complementary purposes.21

First, it can serve as a reminder of the wider dimensions of wellbeing that policy-advisers should take into account in formulating policy advice.

Second, hopefully in time as appropriate models are built and the public’s preferences are able to be more rigorously ascertained and weighted, it can serve as a guide for the quantitative assessment of the tradeoffs between alternative policy options and outcomes [Au et al (2015) and Benjamin et al (2014)].

Third, conditional on having a shared vision of how we wish to live as a society, it can serve as a framework and guide for designing policies that may get us there.

It is the third purpose that is the most exciting and that provides the context for my reference to the late, great Sir Paul Callaghan’s vision of New Zealand as a place where talent wants to live. What may that vision look like?

---

21 Conveniently, the dimensions of the LSF can also be used to measure the success of policies, across time and countries, in expanding the “wellbeing frontier”. We have done some work in this area at the New Zealand Treasury [Sadetskaya (2014), Thomson (2013)].
The generic version of that vision is an expanding wellbeing (LSF) frontier over time. The specific features of that vision that are particularly suited for a country like New Zealand comprise a clean, blue and green, safe and prosperous, country where people of different ethnicities, backgrounds and beliefs live in harmony, and are able to apply their individual talents towards improving their wellbeing. Talent (in all its dimensions) is attracted to this country because it offers the potential to reap high material rewards as well as a great quality of life in its broadest sense.

In the absence of deliberate collective action, several tensions may arise in the stylised model. Examples are possible tensions between: poverty and inequity, and social cohesion; immigration and economic growth, and social cohesion; economic growth, and environmental degradation and resource depletion; and economic growth and equity.

A deliberate multi-dimensional strategy that aims to deliver our vision would be one targeted at expanding the wellbeing (LSF) frontier by focussing primarily on the complementarities between policy instruments and outcomes - effected by influencing the "equations of motion" for both the capital assets that comprise comprehensive wealth, and for the public goods that are wrapped around these capital assets. The following represent the ingredients of a plausible policy package (comprising a set of complementary policy interjections), based on the stylised model to be presented in the next section:

- **Potential economic growth** can be increased through investments in economic and social infrastructure (which are components of aggregate economic and social capital, respectively), as well as improving economic, environmental and social resilience; subsidies for skilling and education towards increasing the domestic supply of skilled labour and scientists; and controls and subsidies towards encouraging the immigration of skilled labour and scientists.

- **Sustainability** of comprehensive wealth can be enhanced by influencing the structure of production (favouring "clean" technology), through appropriate R&D, skilling, education and immigration subsidies, a "carbon tax" (i.e. a tax on the use of the "dirty" technology), appropriate pricing of natural resource use, and direct or joint (private-public) investment in environmental regeneration.

- The evolution of **equity** can be influenced by incentivising more investment in training and education, to be able to produce more skilled labour, and scientists and engineers; and in general investing towards getting people to productive work (i.e. "social investment", to use the language of the current
policy framework in New Zealand). Such policies need to be complemented by poverty-reducing measures if we are genuinely concerned about increasing individuals’ opportunities and capabilities to pursue the kinds of lives they wish to live.

- **Social cohesion** can be enhanced by investing in the teaching of different languages and cultures, actively encouraging the mixing of communities, as well as equity-improving and poverty-reducing measures (i.e. investments in social capital).

- Building **resilience** is about enhancing economic, environmental and social resilience to potential systemic shocks. Examples of policy instruments are those that strengthen our social and economic infrastructure as above, with these investments deliberately adopting a resilience lens – e.g. in investing in roads or buildings, making sure that they include a resilience-improving dimension as well; enhancing our bio-security and cyber-security infrastructure; investing in the teaching of different languages or cultures.

These interconnected and complementary policy interventions are jointly targetted at growing, shaping, managing, appropriately distributing capital stocks across society and across generations, and protecting them against systemic risks. As we will show in the formal model in the next section, these policies operate through their influence on the “equations of motion” that define the evolution of capital assets, and of the public goods that are wrapped around these capital assets.

These policy measures are underpinned by technology, by institutions (“rules of the game”), and by cultures embodied in the country’s social infrastructure (e.g. the rule of law, well defined and secure private property rights, well functioning financial markets and institutions). Although these measures offer the potential to enhance our collective wellbeing, whether that potential will be realised or not will depend on the effectiveness and efficiency of the choice of policy instruments and how they are implemented - including their effects on private individuals’ incentives to do the best they can for themselves and their businesses.

The initial values of the dimensions of the LSF, as well as the initial distribution of resources (such as physical and human capital), are “state variables” (i.e. inherited initial conditions for the dynamics of the future). They all evolve endogenously in response to economic, social, and political forces in pursuit of higher individual and communal wellbeing.

To give a flavour of the endogenous dynamic process at work, note that social infrastructure includes the social and political institutions we have inherited. In-
equality of access to resources creates inequality of opportunities and capabilities, and negatively affects social cohesion. Policy responds by leaning against such inequality in an attempt to enhance social cohesion - and hence overall wellbeing. Policy affects inequality through prices and incentives (via technology) - thus generating "good growth" through effects on technology. This in turn enhances social cohesion and thereby overall wellbeing. Thus we generate a virtual upward well-being spiral. This endogeneity is given effect to by a representative government and the democratic process.

The government funds all its interventions through a tax on the production of the single output. In doing so, it is aware of the complementarities and tradeoffs between the costs and benefits of its interventions on overall wellbeing; its purpose is to enhance overall intergenerational wellbeing.

### 3 A Unified and Stylised Formal Model

This section presents a mathematical version of the narrative provided in Section 3 above. To serve our aspirations, the model needs to capture all key features of the LSF - all capital stocks and all dimensions (i.e. critical public goods, and associated positive externalities, surrounding the capital assets) of the LSF - and connect them to overall wellbeing.

As stated in the Introduction, the components of the stylised model are spread across the existing literature. I simply borrow from this literature, as summarised and enhanced by Acemoglu et al (2012), Acemoglu et al (2014), Arrow et al (2012, 2013), Chichilnisky (1997), Jones and Vollrath (2013), Krugman (1987), and Turnovsky and Mitra (2013), and attempt to expand on and integrate key components of the models presented in these papers, to build a unified stylised model that represents the essence of the LSF.

To repeat a critical point made earlier, a fundamental assumption underpinning the stylised model is that, in addition to their individual incomes and consumption of private goods (including leisure and good health), individuals and businesses also value (i.e. receive positive benefits from) the positive externalities associated with, among others, a clean environment, sustainable comprehensive wealth, social cohesion, equity across society and generations, and enhanced resilience to the types of shocks that have the potential to cause serious damage to various capital
assets and through that on our overall wellbeing.

In terms of presentation style, I do not support all statements made throughout the remainder of this paper with formal proofs, nor do I make an effort to always identify the initial sources of the main ideas. I refer instead to the literature where these formal proofs are provided, and the original sources of the main ideas are cited. My purpose is to show that a formal model for the LSF can be constructed, and this model used to identify the domain of a public policy that aims to enhance intergenerational wellbeing, highlight the key complementarities and tradeoffs that we face as a society in this pursuit, and explore the policy options and levers available to the policy makers to relax these tradeoffs and exploit the complementarities to the same end. Along the way, I do present some key results to highlight how the dynamics of the model works, and to show the channels through which policy options that are suggested will make a difference for sustainable wellbeing.

3.1 Wellbeing, Consumption, Income and Wealth

All individuals, however they generate income [as skilled or unskilled workers, scientists (incl engineers)/researchers, entrepreneurs (producers or importers of machines), producers of the final good], are intergenerational wellbeing maximisers. Wellbeing is a positive function of comprehensive consumption, which in turn is sourced from comprehensive wealth.

3.1.1 Generic Model

The generic version of the basic model is outlined in Arrow et al. (2012) and takes the following form.\textsuperscript{23}

Time is continuous and is denoted by $s$ and $t$ ($s \geq t \geq 0$). The horizon is taken to be infinite.

For our purposes what matters is that the “typical individual’s” wellbeing depends on “comprehensive consumption”; in that particular sense, preferences are identi-

\textsuperscript{22} This sub-section borrows heavily from ideas presented in Arrow et al. (2012, 2013), Acemoglu et al. (2012), Acemoglu et al. (2014), Turnovsky (2013) and Turnovsky and Mitra (2013).

\textsuperscript{23} I reproduce some key derivations of that paper here to set the generic platform for the more specific (stylised) model to be used in the rest of the paper.
The wellbeing function for the “typical individual” (and for society at large) can be written as follows:

\[ W(t) = \int_{t}^{\infty} \left[ U(C(s)) e^{-\rho(s-t)} \right] ds, \rho \geq 0 \]  

(1)

where, \( W(t) \) denotes intergenerational wellbeing at \( t \) and \( C(s) \) denotes a vector of consumption flows at \( s \). \( C \) includes not only marketed consumption goods, but also various others including leisure, arts, health services, and consumption services supplied by nature - i.e. it represents comprehensive consumption. \( U(C(s)) \) represents the collective utility flow at \( s \). \( \rho \) is the discount rate. Thus, intergenerational wellbeing is the discounted utility flows of current and future generations.

An assessment of wellbeing at \( t \) \( [W(t)] \) requires a forecast of utility beyond \( t \). For this purpose, the relevant forecast at \( t \) is the pair of vector functions \( \{C(s), K(s)\} \) for \( s \geq t \), where \( K(s) \) denotes the stocks of a set of capital assets at \( s \) (i.e., comprehensive wealth). \( K(s) \) provides the sources of comprehensive consumption at \( s \). We assume that the integral in expression (1) converges for the forecast.

Current and future wellbeing depends on collective comprehensive wealth; it also depends on the evolving structure of technology, people’s values and preferences, and institutions. The stock of assets that comprise comprehensive wealth at any moment \( s \) will be determined by the stocks at the immediately preceding moment. By proceeding from moment to moment in this way, the entire future course of capital assets and therefore collective wellbeing will be determined.

Thus, given \( K(t) \), \( K(s) \) and \( C(s) \), and thereby \( U(C(s)) \), are determined for all future times \( s \geq t \). Hence, from expression (1), \( W(t) \) is determined as well. Therefore, we can write:

\[ W(t) = W(K(t), t) \]  

(2)

In expression (2), \( W(t) \) depends directly on \( t \), to reflect the potential impact of a set of time-varying factors such as population growth, technological change, and institutions, in addition to capital assets, that affect intergenerational wellbeing. In the stylised model used below to represent the Living Standards Framework (LSF), I will also add various public goods generating positive externalities, such as equity and social cohesion, to this list.

Defining sustainable intergenerational wellbeing as \( dW(t)/dt \geq 0 \) and assuming \( W(t) \) is differentiable in \( K \), a criterion for sustainable intergenerational wellbeing...
at \( t \) can be specified as follows - where capital assets are indexed by \( i \):

\[
dW(t)/dt = \partial W/\partial t + \sum_i \left[(\partial W(t)/\partial K_i(t))(dK_i(t)/dt)\right] \geq 0
\] (3)

We now define two "shadow prices", which will in turn help us define "comprehensive wealth". The first, \( \lambda_{K_i}(t) \), is the (spot) shadow price of the \( i^{th} \) capital asset at time \( t \):

\[
\lambda_{K_i}(t) \equiv \partial W(t)/\partial K_i(t), \text{ for all } i
\] (4)

and represents the contribution to \( W(t) \) made by \( K_i(t) \) both through the goods and services it helps produce, as well as through direct enjoyment of the stock itself. A forest, clean water, and health are three examples. Where there are externalities involved (associated with a "tragedy of commons" for example), an asset’s shadow price can be negative when its market price is positive. "At any date an asset's shadow price is a function of the stocks of all assets. Moreover, the price today depends not only on the economy today, but on the entire future of the economy. So, for example, future scarcities of natural capital are reflected in current shadow prices of all goods and services. That means that shadow prices are functions of the degree to which various assets are substitutable for one another, not only at the date in question, but at subsequent dates as well. Of course, if the conception of intergenerational wellbeing involves the use of high discount rates on the wellbeing of future generations (i.e. if \( \rho \) is large), the influence on today's shadow prices of future scarcities would be attenuated. Intergenerational ethics plays an important role in the structure of shadow prices" [Arrow et al (2012), pp. 323-4].

The second shadow price we need, to define comprehensive wealth, is the shadow price of time which, based on expression (2), we can conceptualise as an additional form of capital asset:

\[
\lambda_T(t) \equiv \partial W/\partial t
\] (5)

We can now use these two shadow prices as weights to construct an aggregate index of the society’s comprehensive wealth \( (V) \):

\[
V(t) = \lambda_T(t)t + \sum_i \lambda_{K_i}(t)K_i(t)
\] (6)

A society's comprehensive wealth is the (shadow) value of all its capital assets [Arrow et al (2012), p. 324].

There is an equivalance relationship between changes in comprehensive wealth
at constant prices and changes in intergenerational wellbeing:

\[ \Delta W(t) = \lambda_T(t) \Delta t + \sum_i \lambda_{K_i}(t) \Delta K_i(t) = \Delta V(t) \]  \hspace{1cm} (7)

or, letting \( I_i(t) = \frac{\Delta K_i(t)}{\Delta t} \), we can establish an equivalence relationship between changes in intergenerational wellbeing and "comprehensive investment" (or "genuine saving"), which is the expression on the right-hand side of the following expression:

\[ \Delta W(t) = \lambda_T(t) \Delta t + \sum_i \lambda_{K_i}(t) I_i(t) \Delta t \]  \hspace{1cm} (8)

It is very important to note that the term "investment" here is used in a wider sense than its typical use, which refers to accumulation, when the only type of capital is "reproducible capital". When reference is to a broader set of capital assets comprising comprehensive wealth, including natural and human capital, "investment" means "any increase in the flow of services that the asset can provide over its lifetime. To leave a forest alone so that it can grow is in our extended sense to invest in the forest. ... To give food to someone hungry ... increases her future productivity - which means that to prevent hunger is to invest in human capital" [Arrow et al (2013), p. 513].

A couple of other points are also worth highlighting before we proceed. First, the ratios of shadow prices are marginal social rates of substitution among the various capital assets, and when \( W(t) \) is maximised these marginal rates of substitution equal their corresponding marginal rates of transformation. Second, sustainability and optimality are different concepts. It is quite possible that along an optimum path, one that maximises \( W \), \( W(t) \) may decline for a while (thus violating the sustainability criterion), and then start increasing again.

3.1.2 Stylised Model

The stylised model that we use in the rest of the paper borrows several key features from the Arrow et al (2012) model outlined above. Most importantly, the object of interest remains intergenerational wellbeing and its sustainability, wellbeing is positively influenced by comprehensive consumption, and comprehensive wealth is the source of comprehensive consumption.

We build on this platform, using the discrete-time model of Acemoglu et al (2012) as a benchmark, but enhancing it through an integrated framework that brings together social, economic and environmental spheres, and associated externalities, as essential and complementary ingredients of wellbeing.
The stylised model embodies the point made by Friedman (2006) that, “the value of a rising standard of living lies not just in the concrete improvements it brings to how individuals live but in how it shapes the social, political and, ultimately, the moral character of a people. Economic growth - meaning a rising standard of living for the clear majority of citizens - more often than not fosters greater opportunity, tolerance of diversity, social mobility, commitment to fairness, and dedication to democracy” (p. 15). However, “economic progress needs to be broadly based if it is to foster social and political progress” (p. 16); economic growth cannot be sustained unless it is socially and environmentally sustainable [Duraiappah and Munoz (2012)]; and relying on market forces alone will not achieve all that.

In addition, the model captures the fundamental notion that the best way to build shared wellbeing on a sustainable basis is to provide individuals with the opportunities and capabilities (embodied in components of comprehensive wealth) to participate productively in economic and social life, so that they can get their fair share of the prosperity they help create - because individual wellbeing is enhanced through active contribution, rather than being passive recipients of welfare payments [Engelbrecht (2015)].

Comprehensive Consumption  Comprehensive consumption refers to the consumption of a set of private and public goods and services. In addition to deriving utility from the consumption of a marketed product (the single output $Y$ in the stylised model), individuals also derive direct utility from the consumption of time (i.e. "leisure") and good health. These are the private goods that enter into the individuals’ utility functions.

In addition, we assume that individuals also benefit from having access to public goods (and associated positive externalities) arising from a high-quality (pristine) environment ($E$), social cohesion ($S$), and increased resilience ($\Gamma_{ES}$) of both ($E$) and ($S$) to potential major systemic shocks:

$$W_0 = \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} \times u(C_y,t, C_{x,t}, C_{h,t}; E_t, S_t, \Gamma_{ES,t})$$

where, $C_y$ is the consumption of the single marketed product, $C_x$ is the consumption of leisure, $C_h$ is the consumption of good health, and $E, S, \Gamma_{ES}$ are all indices $\in (0, 1)$, representing the degree of environmental quality, social cohesion, and the quality and effectiveness of measures that enhance environmental and social resilience to potential major shocks to each, respectively.

Individuals take $E_t, S_t, \Gamma_{ES,t}$ as given in any period $t$, in deciding how to allocate
their income (between the marketed consumption good and saving), and their time (between work, leisure, education or training, and health-enhancing activities). As I show below, investment in education, training and health are different forms of human-capital investment. They may lead to an increase in wellbeing both directly, by providing higher utility, but also indirectly, by increasing productivity and therefore market-income.

The instantaneous utility function \( u(C_y, C_x, C_h; E, S, \Gamma_{ES}) \) is twice-differentiable, increasing and jointly concave in \((C_y, C_x, C_h, E, S, \Gamma_{ES})\). We assume that:

\[
\lim_{C_i \downarrow 0} \frac{\partial W(C_y, C_x, C_h; E, S, \Gamma_{ES})}{\partial C_i} = \infty; \; i \in \{y, x, h\} \tag{10}
\]

\[
\lim_{j \downarrow 0} \frac{\partial W(C_y, C_x, C_h; E, S, \Gamma_{ES})}{\partial j} = \infty; \; j \in \{E, S, \Gamma_{ES}\} \tag{11}
\]

\[
\lim_{j \downarrow 0} W(C_y, C_x, C_h; E, S, \Gamma_{ES}) = -\infty; \; j \in \{E, S, \Gamma_{ES}\} \tag{12}
\]

The last two conditions (11 and 12) highlight and emphasise the point that if the quality of the environment, the degree of social cohesion, and/or aggregate resilience to potential severe shocks to both of these public goods, were to approach their lower bounds this would have severe negative wellbeing consequences.

**Comprehensive Wealth** Comprehensive consumption is sourced from comprehensive wealth which, in the stylised model, comprises some private and some public capital assets. The private capital assets are physical capital (in the form of the machines used to manufacture the single consumable good) and human capital (in the forms of skills, education and health). Public capital assets on the other hand comprise some forms of natural capital, social infrastructure and economic infrastructure. From an overall wellbeing perspective, we are very interested in the evolution of all forms of capital assets, private and public, widespread access to public assets, and the distribution of private capital assets. In the formal stylised model, the evolution of capital assets all have the same structure - investment terms that generate or augment these assets and depreciation terms that deplete them.

All private *physical capital* is owned by individuals (or households), and is augmented through private saving (= investment) and depreciates through use in production. *Human capital* is augmented through investment in education and training, as well as in one’s health, and (in the stylised model) depreciates through pollution (i.e., the degradation of the natural environment).

In the pursuit of higher wellbeing, individuals face two types of resource constraints:
time and market-income.

In the stylised model, time is the resource that can be partly used to accumulate human capital (through training, education and investment in good health), and income is the resource that can be partly used to accumulate physical capital (through saving). Thus human capital is accumulated by investing time and physical capital is accumulated by investing income.

In each period, individuals are all endowed with one unit of time that can be allocated to leisure ($X$) or non-leisure ($NX$) activities. Non-leisure activities comprise working ($L$), investing in training ($TR$) or education ($ED$) (but not both), and investing in one’s health ($H_t$):

$$\begin{align*}
1 & = C_{x,t} + NX_t \\
NX_t & = L_t + TR_t + ED_t + H_t
\end{align*}$$

where, $C_{x,t}$ refers to the consumption of leisure-time, and $L_t, TR_t, ED_t, H_t$ are each $\in [0, 1)$. Thus equations (13) represent the individuals’ time-budget constraints.

Investment in skills-training is required to convert investment-time ($NX - L$) into skilled labour ($L_s$), and investment in education is required to convert investment-time into scientists ($Sc$). In the stylised model, for simplicity, we assume that it takes one period of investment (with the length of the period to be chosen conveniently to suit our purpose) to convert investment-time into skilled labour (through training) or into a scientist (through education). Investment in health is simply a function of time (say the use of time to exercise). Again for simplicity, we assume that investment in health is simply a function of time (say the use of time to exercise).

We assume that these conversions take the following forms:

$$\begin{align*}
L_{s,t+1} &= (1 - \xi_{L_s}) \times L_{s,t} + (TR_t \times \mu_{tr}) \times (NX_t - L_t) \\
Sc_{t+1} &= (1 - \xi_{Sc}) \times Sc_t + (ED_t \times \mu_{ed}) \times (NX_t - L_t) \\
\tilde{H}_{t+1} &= (1 - \xi_{\tilde{H}}) \times \tilde{H}_t + (H_t \times \mu_h) \times (NX_t - L_t) \times \upsilon(E_{t+1})
\end{align*}$$

where, $\tilde{H} \in (0, 1)$ is a "health index", $\partial \upsilon/\partial E > 0$, and $\mu_{tr}, \mu_{ed},$ and $\mu_h$, which are all $\geq 0$, refer to the returns from the investment of time into training, education and health activities, respectively; and $\xi_j > 0$ represents the rate of depreciation of $j$, for $j \in \{L_s, Sc, H\}$. Thus, the evolution of "good health" reflects both private time-investment in health and the impact of the quality of the natural environment on health.

From now on, we will use the term "scientists" to include engineers as well.
In order to be able to consume the marketed product, the individual has to generate market income, which is either spent on consuming the marketed product or saved (= invested). Individuals can generate market income by working as (unskilled or skilled) labourers, doing research (as scientists), working as an entrepreneur, or producing and selling the final product, as well as through their earnings on their ownership of physical wealth (i.e. the machines that are used to produce the final output). Thus an individual’s personal income is measured by the sum of income from physical wealth, and from skilled or unskilled labour, or from research (as a scientist). We can therefore represent the individuals’ income-budget constraints, all expressed in units of the final output, as follows:

\[
\begin{align*}
  r_{K_t}^L K_{p,t}^L + w_t L_t - C_{y,t}^L &= S_a^L 	ag{17} \\
  r_{K_t}^L K_{p,t}^L + w_s L_s,t - C_{y,t}^L &= S_a^L_s 	ag{18} \\
  r_{K_t}^L K_{p,t}^S c + \Pi_t - C_{y,t}^S &= S_a^S_c 	ag{19}
\end{align*}
\]

where \( r_{K_t} \) is the return to physical capital (\( K_p \)) using the existing technology, \( w_s \geq w \) is the market wage of skilled labour (with \( w \) being the market wage of unskilled labour), \( \Pi \) is the profit earned by the successful scientist (see more on this below), and \( S_a \) is saving out of market income. The opportunity cost of working (to be able to increase the consumption of marketable consumer products) is the leisure time that has to be given up, or the loss of time that can be invested in health.

The reason we do not have the budget constraint of entrepreneurs listed separately is because all the profits of the entrepreneurs are expropriated by successful scientists through the competitive process (more on this below). In effect, in the stylised model entrepreneurs are the successful scientists, with the unsuccessful ones generating their income as skilled labour. We also do not have a separate budget constraint for the sellers of the final product because their profit is effectively allocated to households through return on physical capital.

The evolution of aggregate physical capital (machines), or aggregate private physical wealth, measured in units of the final output, can be described by the following set of equations:

\[
\begin{align*}
  S_a t &= S_a^L + S_a^L_s + S_a^S_c 	ag{20} \\
  K_{p,t} &= K_{p,t}^L + K_{p,t}^L_s + K_{p,t}^S_c 	ag{21} \\
  K_{p,t+1} &= K_{p,t} + S_a t - \xi_{K_p,t} K_{p,t} = (1 - \xi_{K_p,t}) K_{p,t} + S_a t 	ag{22}
\end{align*}
\]

where \( \xi_{K_p} \in (0, 1) \) represents the rate of depreciation of physical capital.
Finally, the evolution of aggregate human capital, or the value of aggregate private human wealth, is described by the following equation:

\[ \frac{\lambda_{K_{h,t+1}} K_{h,t+1}}{\frac{\lambda_{L_{s,t+1}} L_{s,t+1} + \lambda_{S_{c,t+1}} S_{c,t+1}}{\lambda_{H_{t+1}}} = \frac{\lambda_{L_{s,t+1}} L_{s,t+1}}{\lambda_{S_{c,t+1}} S_{c,t+1}}} \]

where \( \lambda_j, j \in \{K_{h,t+1}, L_{s,t+1}, S_{c,t+1}\} \) represent the respective spot shadow prices of these various forms of human capital (or human wealth), and \( L_{s,t+1} \) and \( S_{c,t+1} \) are the aggregated versions of equations (14) and (15).

### 3.2 Production and Wellbeing

Comprehensive wealth affects wellbeing both through the direct influences of capital assets (such as a pristine environment and good health) on individuals’ utility, and also indirectly through the production of consumable goods and services, which uses capital assets as inputs, which again enter individuals’ utility functions.

#### 3.2.1 Generic Model

Voosholz (2014) provides a very useful summary of alternative specifications for production functions that link comprehensive wealth to production. Essentially there are four main types: Cobb-Douglas (CD), Constant Elasticity of Substitution (CES), Variable Elasticity of Substitution (VES), and Leontief. Applications vary not only in terms of the type of production function that is used, but also viz the choice of capital assets that are represented in comprehensive wealth. These choices are not inconsequential; they do influence the outcomes of the analysis.

The general version of the production function used in the endogenous growth literature takes the following form:

\[ Y = F(K_p, L, K_h, A, \tilde{R}, \tilde{Z}, K_s) \]

where, \( K_p \) is physical (human-made) capital (or machines) used in production, \( L \) is (unskilled) labour, \( K_h \) is human capital, \( A \) represents technological change, \( \tilde{R} \) represents the use of the non-renewable (exhaustible) resource \( (R) \) and \( \tilde{Z} \) the use of the renewable resource \( (Z) \) in production, and \( K_s \) is social capital.

The inclusion of social capital \( (K_s) \) in the production function can be motivated in various ways. From an economic perspective, we have conceptualised social capital as generalised trust and its networks, enabling cooperation and information
sharing between innovators. Thus one way in which social capital can have an impact on economic growth is by increasing innovation capacity, and through that productivity growth [Agenor and Neanidis (2015), Algan and Cahuc (2013), Thompson (2015)].

The model is then closed by formulating a set of equations of motion that specify how each component of comprehensive wealth evolves over time through a set of influences that lead to their depletion or depreciation, and regeneration.

In what follows, we use the CES specification primarily because of our focus on emphasising whether key inputs into our production, utility, wellbeing and externality functions are complements or substitutes; the CES function provides a very convenient way of making this differentiation, between complementarity and substitutability.

3.2.2 Stylised Model

The stylised model is primarily based on the one used by Acemoglu et al (2012), but also influenced by the discussion in Acemoglu et al (2014). One of the main differences is that our stylised model represents a small open economy, whereas the Acemoglu et al papers work with a closed economy, model.

There is a single, consumable and internationally tradeable, final good ($Y$), produced competitively using two types of technologies, "clean" (or new) and "dirty" (or old), according to the following aggregate production function:\(^{(25)}\)

$$Y_t = \left[ \left( (Ls \tilde{H})^{1-\alpha} \int_0^1 A_{ci}^{1-\alpha} m_{ci}^\alpha di \right)^{\theta_y \frac{1}{\theta_y}} + \left( \tilde{R}^{\alpha_2 \theta_y \frac{1}{\theta_y}} (L \tilde{H})^{1-\alpha_1} \int_0^1 A_{di}^{1-\alpha_1} m_{di}^{\alpha_1} di \right)^{\theta_y \frac{1}{\theta_y}} \right]^{\theta_y \frac{1}{\theta_y}} \right]^{\theta_y \frac{1}{\theta_y}}$$

These different technologies are embedded in the two types of machines used in producing the single output. \(\theta_y \in (0, +\infty)\) in equation (25) is the elasticity of substitution (in production) between the two technologies; \(\tilde{R}\) refers to the quantity of the exhaustible natural resources \(R\) used with the "dirty technology"; \(m_d\) and \(m_c\) refer to the continuum (indexed by \(i\)) of machines (physical capital) using the "dirty" and "clean" technologies, respectively; and \(A_{ci}\) and \(A_{di}\) correspond to the productivity (or "quality") of these machines.

It is assumed that the (health-adjusted) labour required to operate the machines

\(^{25}\) To repeat, the term "dirty technology" is not used in a derogatory sense, but simply as a convenient means of differentiating between two types of technology with significantly different impacts on the accumulation of human capital and the preservation of natural capital.
using the "dirty technology" ($L\tilde{H}$) is "unskilled" (or comparatively less skilled), whereas the (health-adjusted) labour required to operate the "clean technology" ($L_s\tilde{H}$) is "skilled" (or comparatively more skilled). Note that labour is not indexed by $i$; $L_s$ and $L$ can operate all versions of $m_c$ and $m_d$, respectively, equally efficiently.

The two technologies are referred as (gross) substitutes when $\theta_y > 1$ and (gross) complements when $\theta_y < 1$. Reflecting the assumption, and expectation, that, over time, successful "clean technologies" will substitute for "dirty technologies", we assume, in our stylised model, that $\theta_y > 1$. Throughout we ignore the Cobb-Douglas case of $\theta_y = 1$. $\alpha, \alpha_1, \alpha_2 \in (0,1)$, $\alpha_1 + \alpha_2 = \alpha$.

The evolution of the exhaustible resource ($R$) is given by:

$$R_{t+1} = R_t - \tilde{R}_t \quad (26)$$

The per unit extraction cost for the exhaustible resource, expressed in units of the final output ($Y_t$), is $c(R_t)$, where $c$ is a non-increasing function of $R$.

### 3.3 Technology and Market Structure

In line with the literature on endogenous technical change [Jones and Vollrath (2013)], the production side of the economy consists of three sectors: a final-goods sector, an intermediate-goods sector, and a research sector.

The single final output ($Y$) is consumable and internationally tradeable; it is produced using the process described by the production function in equation (25). Although our small-open economy is a price-taker in the global market for this good, there is an increasing price premium on this product as the weight of "clean technology" increases in its production.

The machines using both technologies are supplied by monopolistically competitive firms. These machines may be manufactured domestically or imported from overseas. This represents the intermediate-goods sectors.

The research sector, which is comprised of scientists, generates new ideas. These scientists may be domestically-educated or may be immigrants from overseas. Some of these ideas lead to productivity improvements in machines using the "dirty technology", whereas others lead to further productivity improvements in the "clean technology".

---

This productivity term is referred to as \( A \) in equation (25), and we define the average productivity of process, or machine type, \( j \in \{c, d\} \) as:

\[
A_{jt} \equiv \int_0^1 A_{jit} \, di
\]  

(27)

which implies that \( A_{dt} \) corresponds to "dirty technologies" whereas \( A_{ct} \) relates to "clean technologies". For concreteness, "clean innovation", that is increasing \( A_{ct} \), can be thought of as reducing the pollution, or the exhaustible natural resource, intensity of the overall production process.

\( A_{jt} \) evolves over time according to the following difference equation:

\[
A_{jt} = (1 + \mu \eta_j S_{cj}) \times A_{jt-1}
\]  

(28)

where, \( S_{cj} \) references the group of scientists working with technology \( j \), \( \eta_j \in (0, 1) \) is the probability that they will be successful in innovation, and \((1 + \mu) \) (where \( \mu > 0 \)) is the factor by which innovation increases the quality (or productivity) of a machine when the scientists are successful in innovating.

At the beginning of each period, each scientist decides whether to direct his/her research to improving the quality of machines using "clean" or "dirty" technology. She/he is then randomly allocated to at most one machine (without any congestion; so that each machine is also allocated to at most one scientist). A successful scientist, who has invented a better version of machine \( i \) using technology \( j \in \{c, d\} \), obtains a one-period patent and becomes the entrepreneur for the current period in the production or importation of machine \( i \). In cases where innovation is not successful, monopoly rights are allocated randomly to an entrepreneur drawn from the pool of potential entrepreneurs, who then uses the old technology. This "innovation possibilities frontier" where scientists can target only a technology or machine type (rather than a specific machine) ensures that scientists are allocated across the different machines using a particular technology.

### 3.4 Distribution and Equity\(^{27}\)

As I explained in Section 3.3 above, it is the distribution of opportunities and capabilities among individuals, in a way that gives them a fair go at pursuing the kinds of lives they have reason to value, that is at the core of our concept of equity. Access to public forms of capital (such as social and economic infrastructure, and some forms of natural capital), the allocation of private wealth (in the form

\(27\) This Section of the paper draws heavily on Turnovsky (2013) and Turnovsky and Mitra (2013).
of the ownership of physical capital and human capital), the evolution of the production technology used in producing the final output, and public policy that has an influence on both the quantum of and access to public goods that generate positive externalities, all have a major bearing on equity defined in this way. In our stylised model, increasing equity has a direct positive benefit on wellbeing through enhanced social cohesion. In addition, policies that incentivise skilling, innovation-supporting education and R&D, as well as the switch of production technology towards "clean technology", all have positive benefits for both equity, and the quality and sustainability of potential economic growth.

In order to be able to discuss and present distributional aspects or equity across society, we need to differentiate between individuals. In our stylised model, we differentiate between three groups of individuals - unskilled labourers, skilled labourers, and scientists/entrepreneurs. In addition to using their non-leisure time in different ways, to generate income or to invest in building human capital, these groups of individuals also differ in their initial (period 0) endowments of human capital \( \{ K^L_{h,0}, K^L_{s,0}, K^L_{Sc,0} \} \) and physical capital \( \{ K^L_{p,0}, K^L_{p,0}, K^L_{Sc,0} \} \) (comprising the machines referred to earlier):

\[
K_{p,0} = \int_0^1 m_{c,i,0}di + \int_0^1 m_{d,i,0}di = K^L_{p,0} + K^L_{p,0} + K^L_{Sc,0}
\]

\[
\lambda_{K_{h,0}K_{h,0}} = \lambda_{L_{s,0}L_{s,0}} + \lambda_{Sc,0}Sc_0
\]

where \( \lambda_j, j \in \{ K^L_{h,0}, L_{s,0}, Sc_0 \} \) represent the respective spot shadow prices of these various forms of human capital (or human wealth). These two components of private capital represent the accumulated, initial or period 0, collective private capabilities and opportunities of these groups of individuals.

Ultimately, we are interested in the distribution of wellbeing. Wellbeing is a function of comprehensive consumption, and comprehensive consumption is a function of comprehensive wealth. Comprehensive wealth comprises public capital (comprising components of natural capital and social capital) and private capital (a subset of economic capital, as well as human capital). The distribution of comprehensive wealth, which is a primary determinant of the distribution of wellbeing, is thus a function of access to public capital assets and ownership of private capital assets.

With that context and background in mind, let us start with the initial distribution of total private human capital \( K_{h,0} \) (which comprises the skills, education and health of individuals) and total private physical capital \( K_{p,0} \) (comprising either type of machine). In a growing economy, we are interested in the evolving shares of

28 In the stylised model, successful scientists become entrepreneurs; and unsuccessful ones generate income as skilled labour.
these three groups of individuals in the accumulating total stocks of human capital, $k_{h,t}^j = K_{h,t}^j / \bar{K}_{h,t}$, $j \in \{L_s, Sc\}$ and physical capital, $k_{p,t}^j = K_{p,t}^j / \bar{K}_{p,t}$, $j \in \{L, L_s, Sc\}$, where $\bar{K}_{h,t}$ and $\bar{K}_{p,t}$ denote the corresponding economy-wide average quantities.

The initial relative endowments across these three groups of individuals have mean one, standard deviations $\sigma_{k_h,0}$ and $\sigma_{k_p,0}$, measured around the economy-wide averages, and covariance (or in this case, coefficient of variation) $\sigma_{k_h,k_p,0}$ (possibly zero).

At the beginning of each time period $t$, the total private wealth ($V$) of the three groups of individuals comprises their physical capital plus the value of their human capital, and is defined by:

$$V_t^L = \lambda_{K_p,t} K_{p,t}^L$$  \hfill (31)

$$V_t^{L_s} = \lambda_{K_p,t} K_{p,t}^{L_s} + \lambda_{L_s,t} L_{s,t} \bar{H}_t$$  \hfill (32)

$$V_t^{Sc} = \lambda_{K_p,t} K_{p,t}^{Sc} + \lambda_{Sc,t} Sc_t \bar{H}_t$$  \hfill (33)

where $\lambda_i, i \in (K_p, L_s, Sc)$ are the shadow prices of physical capital, skilled labour and scientists.

Wealth accumulation at constant prices is the sum of physical capital accumulation (a function of saving and investment out of market income) and human capital accumulation using time (in the form of investment in skilling, education and health).

We define the share of total private wealth owned by each of the three clusters of individuals as $v_j \equiv V^j / V$, $j \in (L, L_s, Sc)$.

It is quite obvious from equations (25), (29)-(33) that the evolution of wealth inequality over time will reflect the evolution of the structure of the economy (i.e. the composition of production), and the values of the various types of physical and human capital.

Using a similar, but not identical, model Turnovsky and Mitra (2013) show that an individual who has above average long run (equilibrium) wealth also enjoys above average long run consumption and thus wellbeing.

We can then define the degree of inequality in society by the degree of inequality of opportunities and capabilities (and thereby of wellbeing) across the three clusters of individuals, which in turn reflects the standard deviation of wealth around average wealth ($\sigma_{v,t}$):

$$Q_{t+1} \equiv \sigma_{v,t}$$  \hfill (34)
where \( Q \) is an index of inequity. An important result of the analysis of Turnovsky and Mitra (2013) is that, in long-run equilibrium, \( \tilde{\sigma}_{v,t} > \tilde{\sigma}_{y,t} > \tilde{\sigma}_{w,t} \) (where \( \tilde{\cdot} \) above a variable indicates its long-run equilibrium value). That is, wealth inequality tends to overstate income inequality, which in turn tends to overstate wellbeing (or welfare) inequality.

\( Q \) can converge towards zero around high or low aggregate average wealth (and thus wellbeing) levels. In our stylised model, everything else being equal, it is convergence of inequity to zero around high levels of wealth that would be consistent with higher levels of overall wellbeing. The aim of policy is to ensure a greater degree of equality of opportunities and capabilities (and thereby of wellbeing) across groups of individuals, without compromising overall wellbeing.

As we will argue later in this paper, this can be achieved with the help of policies that simultaneously support the switch of production towards the use of “clean” technology (through a combination of subsidies and taxes), while concurrently (through R&D support and related measures) encouraging investment in (and thereby increased supply of) human capital in the form of skilled labour and scientists, as well as “clean” machines. Subsidies for education and skilling can help reduce the inequality in the distribution of human capital. Since this would decrease the inequality in the distribution of market income, it also decreases the inequality in the ownership of physical capital (through the convergence of saving rates across these clusters of individuals). Overall we would have lower \( \sigma_v \), and thus \( Q \). We thus potentially have a combination of policies that can simultaneously increase both the quality and sustainability of the economic growth (i.e. material wellbeing) potential of the economy, while also increasing equity and reducing poverty, thus increasing social cohesion and overall wellbeing.

Note again that equity is a very different concept from poverty. The first is a relative measure (or index) of opportunities and capabilities, and the second an absolute measure (or index) of wellbeing-deprivation. Below we specify them as distinct influences on social cohesion, because they require different types of policy interventions.

### 3.4.1 Externalities, Time Horizons and the Government

**Consumption** In terms of consumers, policies that enhance social cohesion, environmental quality, and social and environmental resilience represent significant sources of positive externalities. We can capture these by re-writing equation (9)
as follows:

\[ W_0 = \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} \times \Omega_{w,t}(E_t, S_t, \Gamma_{ES,t}) \times u(C_{y,t}, C_{x,t}, C_{h,t}) \]  
(35)

where, we assume that \( u(C_{y,t}, C_{x,t}, C_{h,t}) \) takes the following specific form:

\[ u(C_{y,t}, C_{x,t}, C_{h,t}) = h(\tilde{H}_t) \times \left[ (C_{y,t})^{\frac{\theta_{\omega} - 1}{\theta_{\omega}}} + (C_{x,t})^{\frac{\theta_{\omega} - 1}{\theta_{\omega}}} \right]^{\frac{\theta_{\omega}}{\theta_{\omega} - 1}} \]  
(36)

where, \( \tilde{H}_t \equiv C_{h,t}, \partial h/\partial \tilde{H} > 0, h(\tilde{H}_t) \in (0, 1) \), and \( \Omega_{w,t}(E_t, S_t, \Gamma_{ES,t}) \in (0, 1) \) represents the wellbeing-externality function. \( \theta_c \) is the elasticity of substitution, in the provision of utility, between the marketed consumer product and leisure; and we assume these to be gross substitutes (i.e. \( \theta_c > 1 \)).

To re-emphasise a point I made earlier in relation to equations (11) and (12), what equations (35)-(36) say is that we enjoy life, including consumption of marketed products, leisure, and our good health, more when we live in a country that offers relatively higher levels of environmental quality, social cohesion, and resilience to potential threats to both; and as our health deteriorates, we derive less and less utility from consuming marketed consumer products and/or leisure. We assume that \( \Omega_w(E, S, \Gamma_{ES}) \) takes the following specific (CES) form:

\[ \Omega_w(E_t, S_t, \Gamma_{ES,t}) = \left[ (E_t)^{\frac{\theta_{\Omega_w} - 1}{\theta_{\Omega_w}}} + (S_t)^{\frac{\theta_{\Omega_w} - 1}{\theta_{\Omega_w}}} + (\Gamma_{ES,t})^{\frac{\theta_{\Omega_w} - 1}{\theta_{\Omega_w}}} \right]^{\frac{\theta_{\Omega_w}}{\theta_{\Omega_w} - 1}} \]  
(37)

where \( \theta_{\Omega_w} \) represents the elasticity of substitution, in the provision of wellbeing-externalities, between environmental quality, social cohesion, and the infrastructure that supports the resilience of both to systemic shocks; we assume these to be gross complements (i.e. \( \theta_{\Omega_w} < 1 \)).

The equations of motion describing the evolution of \( E, S \) and \( \Gamma_{ES} \) are assumed to take the following very similar forms in the stylised model:

\[
\begin{align*}
E_{t+1} &= -\xi_{E,Y} \times (Y_{dt}/Y_t) \times Y_t + (1 + \delta_{E,t}) \times E_t \\
S_{t+1} &= -\xi_{S,F} \times F_{t+1} - \xi_{S,Q} \times Q_{t+1} - \xi_{S,POV} \times POV_{t+1} + (1 + \delta_{S,t}) \times K_{s,t} \\
\Gamma_{ES,t+1} &= [\gamma_{\Gamma_{E}} \times E_t + (1 + \delta_{\Gamma_{E},t}) \times \Gamma_{E,t}] + [\gamma_{\Gamma_{S}} \times S_t + (1 + \delta_{\Gamma_{S},t}) \times \Gamma_{S,t}]
\end{align*}
\]  
(38-40)

where \( \xi_{E,Y} > 0 \) represents the rate of environmental degradation resulting from the production of the single output (\( Y \)) (which depletes the stock of our exhaustible natural resources), weighted by \( (Y_{dt}/Y_t) \) which is a measure of the extent to which overall production uses “dirty” technology (embedded in “dirty” machines); and \( \delta_E \geq 0 \) is the rate of “environmental regeneration”. This equation introduces the
environmental externality associated with production. Innovations or investments in both $\xi_{E,Y}$ and $\delta_E$ are public goods, and therefore potential targets or instruments for public policy.

Similarly, $\xi_{S,F} > 0$, $\xi_{S,Q} > 0$ and $\xi_{S,POV} > 0$ measure the rates of degradation of our social cohesion, arising from immigration [$F_t$ measuring the number (stock) of migrants at the beginning of period $t$], inequity ($Q_t$), and poverty ($POV_t$) respectively; and $\delta_{K_s} \geq 0$ is the rate of enhancement of social capital. $\gamma_{E}$ and $\gamma_{S}$ are both $> 0$; thus, the higher the levels of environmental quality and social cohesion, the greater their resilience to systemic shocks. $\delta_{\Gamma_E} \geq 0$ and $\delta_{\Gamma_S} \geq 0$ refer to the rates of regeneration or enhancement in environmental and social resilience respectively. These equations collectively introduce the wellbeing-externalities associated with investments in environmental quality and social cohesion, as well as enhanced resilience of both to systemic risks. Innovations in $\xi_{S,F}$, $\xi_{S,Q}$, $\xi_{S,POV}$, $\delta_{K_s}$, $\gamma_{E}$, $\delta_{\Gamma_E}$, $\gamma_{S}$, $\delta_{\Gamma_S}$, are public goods, and therefore potential targets or instruments for public policy.

$POV$ directly and negatively affects social cohesion [equation (39)] and indirectly, through its negative effect on social cohesion, social resilience [equation (40)]. Thus, reducing poverty improves social cohesion, as well as enhancing social resilience (partly by providing a "safety net" for the most vulnerable). It is wellbeing-poverty (and not just income-poverty) that matters for social cohesion. We specify the equation of motion for poverty as follows:

$$POV_{t+1} = (1 - \delta_{POV,t}) \times POV_t - \gamma_{V,POV} \times V_t$$

(41)

where $\delta_{POV} \in (0, 1)$ represents the rate of change of poverty - a potential target or instrument for public policy; and $\gamma_{V,POV} > 0$ the positive effect of an increase in overall wealth on poverty reduction.

**Production** On the production side of the stylised model, policies that improve economic infrastructure, enhance broader economic resilience, and build social capital represent significant sources of positive externalities. We attempt to capture
these particular positive externalities by expanding equation (25) as follows:

\[
Y_t = \Omega_y(\Lambda_{y,t}, \Gamma_{y,t}, K_{s,t}) \times \left\{ \left( (L_t \tilde{H}_t)^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} \, m_{dit}^\alpha \, di \right)^{\frac{\theta_y-1}{\theta_y}} + \left( \tilde{R}_t^{\alpha_s} (L_t \tilde{H}_t)^{1-\alpha_s} \int_0^1 A_{dit}^{1-\alpha_s} \, m_{dit}^\alpha \, di \right)^{\frac{\theta_y-1}{\theta_y-1}} \right\}
\]

where, \( \Omega_y(\Lambda_y, \Gamma_y, K_s) \in (0, 1) \) represents the economic-externality function, with economic (or production) infrastructure (\( \Lambda_y \)), economic (or production) resilience (\( \Gamma_y \)) and social capital (\( K_s \)) being its three main arguments. It is best to think of all three arguments of \( \Omega_y(\Lambda_y, \Gamma_y, K_s) \) as index numbers \( \in (0, 1) \). \( \Omega_y(\Lambda_y, \Gamma_y, K_s) \) is increasing in all of \( \Lambda_y, \Gamma_y \) and \( K_s \), twice-differentiable and jointly concave in \( (\Lambda_y, \Gamma_y, K_s) \). The economy approaches its productive-potential when \( \Lambda_y, \Gamma_y \) and \( K_s \) are all close to 0, and operates significantly below potential when they are close to 1, albeit production remains positive even then. The growth in the economy’s productive potential in turn is primarily driven by the growth in productivity (A).

We assume that \( \Omega_y(\Lambda_y, \Gamma_y, K_s) \) takes the following specific form:

\[
\Omega_y(\Lambda_{y,t}, \Gamma_{y,t}, K_{s,t}) = \left[ (\Lambda_{y,t})^{\frac{\theta_{\Omega_y}-1}{\theta_{\Omega_y}}} + (\Gamma_{y,t})^{\frac{\theta_{\Omega_y}-1}{\theta_{\Omega_y}}} + (K_{s,t})^{\frac{\theta_{\Omega_y}-1}{\theta_{\Omega_y}}} \right]^{-1}
\]

where \( \theta_{\Omega_y} \) represents the elasticity of substitution, in the provision of production-externalities, between economic infrastructure, economic resilience and social capital; we assume these to be gross complements (i.e. \( \theta_{\Omega_y} < 1 \)). The equations of motion describing the evolution of \( \Lambda_y, \Gamma_y \) and \( K_s \) can be written as follows:

\[
\begin{align*}
\Lambda_{y,t+1} & = -\xi_{\Lambda_y,Y} \times Y_t - \xi_{\Lambda_y,F} \times F_{t+1} + (1 + \delta_{\Lambda_y,t}) \times \Lambda_{y,t} \\
\Gamma_{y,t+1} & = (1 + \delta_{\Gamma_y,t}) \times \Gamma_{y,t} + \gamma_{\Gamma_y} \times \Lambda_{y,t+1} \\
K_{s,t+1} & = (1 + \delta_{K_s,t}) \times K_{s,t} + \gamma_{K_s} \times \Pi_t
\end{align*}
\]

where \( \xi_{\Lambda_y,Y} > 0 \) measures the rate of degradation of economic infrastructure, resulting from production activities in this economy, \( \xi_{\Lambda_y,F} > 0 \) represents the pressure on economic infrastructure arising from an increase in the number of migrants (or population growth more generally); and \( \delta_{\Lambda_y} \geq 0, \delta_{\Gamma_y} \geq 0 \) and \( \delta_{K_s} \geq 0 \) are the rates of enhancements to economic infrastructure, economic resilience and social capital respectively. \( \gamma_{\Gamma_y} > 0 \); thus, the stronger economic infrastructure is, the greater its resilience to major systemic shocks. Similarly, \( \gamma_{K_s} > 0 \); thus, the greater the profitability of scientific/entrepreneurial activities, the greater the
incentive to invest in social capital (in the form of building clusters and networks). These equations introduce the economic externalities associated with investments in economic infrastructure, economic resilience and social capital. Innovations in \( \xi_{\lambda, \gamma} \), \( \xi_{\lambda, \gamma} \), \( \delta_{\Gamma, \gamma} \), \( \delta_{\Gamma, \gamma} \), \( \gamma_{\lambda} \), and \( \gamma_{\lambda} \) are public goods, and therefore potential targets or instruments for public policy.

**Government** We observe from equations (35) and (42) that the impact of the wellbeing and economic externality functions are of the Hicks-neutral type; they are multiplicative and wrap around the utility and production functions, respectively. They affect the potential aggregate wellbeing and economic growth that can be achieved, and their evolution is endogenous. A "government" that is interested in enhancing intergenerational wellbeing [either towards optimising intergenerational wellbeing in equation (35) or at least maintaining \( \frac{dW_t}{dt} \geq 0 \)] would have to respond to the evolution of the factors that affect these externalities.

There is one more potential distinctive role for the government, that also provides a positive externality for future generations, namely that it could adopt a longer time horizon than a typical individual or business. To capture this additional role of government in our model, we introduce the following (expanded) version of equation (35), which we refer as the Chichilnisky wellbeing function, in reference to Chichilnisky (1997):

\[
W_0 = \omega \times \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} \times \Omega_w(E_t, S_t, \Gamma_{ES,t}) \times u(C_y,t, C_x,t, C_h,t) + (1 - \omega) \times \lim_{t \to \infty} \Omega_w(E_t, S_t, \Gamma_{ES,t}) \times u(C_y,t, C_x,t, C_h,t)
\]

(47)

where, \( \omega \in [0, 1] \) represents the weight we put on the welfare of future generations vs current generations, and:

\[
\lim_{t \to \infty} \Omega_w(E_t, S_t, \Gamma_{ES,t}) \times u(C_y,t, C_x,t, C_h,t) > 0
\]

(48)

Chichilnisky refers to \( \omega \) as a measure of the tyranny (or dictatorship) of present generations against future generations. The case where \( (1 - \omega) > 0 \) is referred as representing "sustainable preferences", where the preferences of both current and future generations are reflected in the aggregate wellbeing function.

Generically speaking, the solution to the optimisation problem with sustainable preferences will lead to a lower production and consumption path but higher wellbeing than one where \( \omega = 1 \) [see Chichilnisky (1997)].

The government has two sets of policy instruments it can use to affect the aggre-
gate outcomes of economic and social activity: regeneration investments (often in cooperation with individuals, businesses and communities) through which it influences the "equations of motion" for externalities [by adjusting the parameters or instruments \( \delta_j, j \in \{E, S, \Gamma_S, \Gamma_E, \Gamma_y, \Lambda_y, POV, K_s\} \); and behaviour-affecting instruments such as subsidies or penalties (operating through the "equations of motion" of private capital assets), through which it influences, among others, the mix of production, mix of human capital through skilling, education and immigration, mix of research and science, and the quantum and mix of physical capital investments. It pays for all this through a tax on the production of the final output.

We write the government's budget constraint as follows:

\[
\left[ \begin{array}{c}
\Omega_w(t) + \Omega_y(t) + B(t) \\
\Omega_w(t) + \Omega_y(t) + B(t) \\
\Omega_w(t) + \Omega_y(t) + B(t) \\
\end{array} \right] (1 + \chi) = \tau Y_t \tag{49}
\]

for \( t = 0, 1, 2, \ldots \). The first two terms on the left-hand side are the cost functions for the delivery of positive externalities, or reduction of negative externalities, to individuals and businesses, respectively; and \( B \) refers to all government activities (subsidies and penalties) directed at influencing the behaviours of private people, institutions and communities - all measured in units of the final output (\( Y \)). All three cost functions \( c_j(.), j \in \{\Omega_w, \Omega_y, B\} \) are assumed to be non-increasing in their respective arguments. As in Acemoglu et al (2014), the parameter \( \chi \) represents the potential distortionary effects, in terms of lost output, of all three types of government interference with economic and social life, in recognition of the possibility that government activities may lead to wastage. The government pays for its activities through the taxes it imposes on production (\( \tau Y_t \)).

Equation (49) highlights an obvious, but critical, point, namely that we need production (income) to generate the revenue with which we fund the positive externalities and behavioural adjustments that deliberate collective action may lead to, towards improving our overall wellbeing. There are potential costs if we do not get it right. If, in trying to generate and fund these potential benefits, we blunt the desire to work, invest and produce, we risk ending up with lower aggregate wellbeing than we would have achieved with no collective action. Thus the stylised model captures both the direct (\( \tau Y_t \)) and the potential indirect (\( \chi \)) costs of government interference on wellbeing through its effects on production (\( Y_t \)).

In the context of the Living Standards Framework (LSF), it is very important to focus on equations (42) and (47) jointly:

- the four capital stocks that sit in the middle of the LSF are embedded in the production or wellbeing functions, yielding services and wellbeing either
directly, or indirectly through production;

- the dimensions of the LSF surround (or are wrapped around) the production and utility functions as public goods that are potential sources of significant positive externalities; and

- if the values of these externality functions tend towards zero, reflecting a degradation in economic and/or social infrastructure, and/or the quality of the environment, overall wellbeing also tends towards zero.

3.4.2 International Connectedness

The stylised small open economy is connected to the rest of the world through trade of the final good, as well as flows of physical capital, people and ideas.

The final good can be consumed at home, saved (= invested), or exported. Physical capital, in the form of machines used in the production of the final output, can be manufactured at home or imported. People can emigrate or immigrate. Immigrants can be unskilled labourers, skilled labourers (who bring in skills), or scientists (who bring in new ideas).

Ideas or new technology can also be embedded in machinery that is imported from overseas. Imports of machinery are associated with local or overseas entrepreneurs bidding for ideas generated in the small open economy or overseas, with the purpose of implementing them in the small open economy and, if successful, obtaining the monopoly rights for producing or importing the machines embedding the new technology (i.e. new ideas).

People emigrate to the small open economy from their own countries because they are attracted by “the quality of life” there, and/or they obtain higher economic rewards (real wages or return to research) by doing so. In other words, it is economic as well as life style advantages that attract immigrants to this small open economy. In our stylised model, “quality of life” reflects both social cohesion and the quality of the natural environment.

The single, internationally tradeable and consumable, final good can be produced with different mixes of the “clean” and “dirty” technology. The higher the weight

\[^{29}\text{The primary insights for this section come from Krugman (1979).}\]

\[^{30}\text{What the stylised model does not allow for is the direct export of ideas or technology developed in the small open economy. In fact, this may be the most effective way of “scaling up” ideas developed in the small open economy, by selling the right to use them in any suitable applications around the world. In the New Zealand context, two examples where this potential exists are agricultural technology and health technology.}\]
of the "clean technology" in producing the good, the higher the price-premium (quasi-rent) the producers earn from the sale of the good on both the domestic and international markets. This in turn has a positive influence on both the skilled labour used to operate the "clean technology" (through higher real wages) and the return on the machines (physical capital) that embody the "clean technology". This positive differential is a reward to international skilled labour and "clean machines" as well, which attracts both factors of production to the small open economy.

We capture these insights through the following equations:

\[
F_{t+1}^L = (1 - \xi_{F^L_t}) \times F^L_t + F^L(S_{t+1}/\bar{S}, E_{t+1}/\bar{E}, w_t/\bar{w}) \quad (50)
\]

\[
F_{t+1}^{Ls} = (1 - \xi_{F^{Ls}_t}) \times F^{Ls}_t + F^{Ls}(S_{t+1}/\bar{S}, E_{t+1}/\bar{E}, w_s^s/\bar{w}^s) \quad (51)
\]

\[
F_{t+1}^{Sc} = (1 - \xi_{F^{Sc}_t}) \times F^{Sc}_t + F^{Sc}(S_{t+1}/\bar{S}, E_{t+1}/\bar{E}, \Pi_t/\bar{\Pi}) \quad (52)
\]

\[
F_{t+1}^{Mc} = (1 - \xi_{F^{Mc}_t}) \times F^{Mc}_t + F^{Mc}(P_y/\bar{P}) \quad (53)
\]

\[
P^{y}_{t}/\bar{P} = P^{y}(Y^d_t/Y_t) \quad (54)
\]

where, \(F^j, j \in \{L, L_s, Sc, Mc\}\) refer to the stocks of foreign unskilled and skilled labour, scientists, and "clean" machines respectively; \(\xi_j \in (0, 1)\) represents the rate of depreciation of \(j\), for \(j \in \{L, L_s, Sc, Mc\}\); \(\Pi\) refers to profits; and a bar over a variable denotes its exogenous "world" counterpart. \(F^j(\cdot), j \in \{L, L_s, Sc, Mc\}\) are all twice-differentiable and increasing in all their arguments; and \(P^{y}(Y^d_t/Y_t)\) is also twice-differentiable but decreasing in its argument \((Y^d_t/Y_t)\).

We note that inward migration is a potential vehicle for changing the composition of our human capital through the inflow of skilled labour, and/or scientists.

In the stylised model, to lift the rate of economic growth we must lift productivity growth. One avenue for this is to increase the number of skilled workers, scientists and engineers, which can partially be achieved by building connections with the rest of the global economy. The return from inward migration of human capital is the positive effect of the quantum and skill-base of migrants to the rate and composition of our economic growth. The cost of inward migration is the (hopefully temporary) pressure it places on social as well as economic infrastructure.
We eventually compare the long-run (steady state) laissez-faire and wellbeing-maximising equilibrium solutions of the model. Doing so helps us first, to demonstrate that the laissez-faire equilibrium of the model is not "socially optimal"; and second, to identify the types of policies that can potentially direct (or incentivise) individuals, communities and businesses towards behaviour that helps us move closer to the socially-optimal outcomes. Towards the second objective, along the way, we present a set of temporary-equilibrium solutions to highlight the key dynamic interactions that public policy can influence.

There is no implication that the government has the ability (including all relevant information) and willingness to solve an intergenerational optimisation problem. Rather, by identifying the gaps and the sources of the gaps between the laissez-faire and wellbeing-maximising solutions of the model, we can highlight where and how public policy can potentially (if implemented effectively and efficiently) make a difference towards improving overall intergenerational wellbeing.

We compare three settings with respect to overall wellbeing, across the five dimensions of the Treasury’s Living Standards Framework: potential economic growth, sustainability, equity, social cohesion, and resilience of the economic, environmental and social systems to major risks or shocks. These are summarised in the Table at the end of this Section. These three settings are represented by two "laissez-faire" models and then a third one (the “wellbeing-maximisation model”) which allows for deliberate public policies towards improving intergenerational wellbeing - funded through a tax on final output.

The first of the two laissez-faire models is essentially the one used by Acemoglu et al (2012) and provides a useful benchmark. It is a closed-economy model and has a smaller set of capital assets and externalities than the model of this paper. The second laissez-faire model is the model of this paper, with no deliberate public policy. The third model is the second model expanded to allow deliberate public policy.
4.1 Actors and Actions

There are three groups of actors in this model: individuals (or households), firms and the government. All individuals, however they earn their market incomes (as unskilled workers, skilled workers, scientists/researchers, entrepreneurs, or final goods producers) are assumed to be intertemporal utility maximising consumers. In addition, there exist two types of firms, those producing the final product and selling it domestically or overseas, and those producing machines or importing them from overseas; they are all profit-maximisers. And finally we have the government, the agent through which individuals organise and coordinate collective action, whose role it is to enhance individual and communal wellbeing.

4.1.1 Utility Maximising Individuals\(^3\)

At the beginning of each period \(t\), we have an existing "mature" stock of unskilled labour, skilled labour, scientists/researchers and entrepreneurs, as well as a new generation of "young" people who join them.

The former ("mature") groups each maximise the following type of intertemporal wellbeing function:

\[
W_0 = \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} \times \Omega_w(E_t, S_t, \Gamma_{ES,t}) \times h(\tilde{H}_t) \times \left[ (C_{y,t})^{\frac{\theta_y - 1}{\theta_y}} + (C_{x,t})^{\frac{\theta_x - 1}{\theta_x}} \right]^{\frac{\theta_c}{\theta_c - 1}}
\]

subject to their respective income-budget constraints represented by equations (17)-(19); the evolution of \(\tilde{H}_t\) as specified in equation (16); the time-budget constraint below:

\[
1 = C_{x,t} + L_t + H_t
\]

and taking \((E, S, \Gamma_{ES})\) as given, by choosing the time paths of \(\{C_{y,t}, C_{x,t}, H_t, L_t, K_{p,t}\}\), for \(t = \{1, 2, 3, \ldots\}\).

On the optimal time path for these choice (or control) variables, the optimality conditions comprise a set of efficient allocation conditions, which essentially state that under an optimal consumption plan, the marginal wellbeing derived from all forms of consumption in period \(t\) is equal to the marginal wellbeing derived from

\(^{3}\) This section is substantially based on Turnovsky (2013) and Turnovsky and Mitra (2013).
consumption in all future periods discounted to the present at the rate $\beta \equiv 1/(1+\rho)$:

$$\Omega_w(\cdot) \times h(\cdot) \times \frac{\partial u(\cdot)}{\partial C_{y,t}} = \beta \times \lambda_{Kp,t+1}$$

(57)

$$\Omega_w(\cdot) \times h(\cdot) \times \frac{\partial u(\cdot)}{\partial C_{x,t}} = \beta \times \lambda_{T,t+1}$$

(58)

$$\Omega_w(\cdot) \times u(\cdot) \times \mu_h \times \tilde{H}_{t-1} \times \nu(E_{t+1}) = \beta \times \lambda_{T,t+1}$$

(59)

a couple of arbitrage conditions:

$$\beta \times \lambda_{T,t+1} = \lambda_{T,t}$$

(60)

$$\beta \times \lambda_{Kp,t+1} \times (r_{Kp,t} - \xi_{Kp,t}) = \lambda_{Kp,t}$$

(61)

and two solvency conditions in the form of income-budget and time-budget constraints:

$$K_{P,t+1} = (1 + r_{Kp,t} - \xi_{Kp}) \times K_{p,t} + w_t \times L_t - C_{y,t}$$

(62)

$$1 = C_{x,t} + L_t + H_t$$

(63)

where, $u(\cdot) = \left[ (C_{y,t})^{\frac{\theta_c-1}{\theta_c}} + (C_{x,t})^{\frac{\theta_c-1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c-1}}$. These optimality conditions, written for $L$, also hold for $L^s$ and $S_c$, with suitable adjustments based on equations (17)-(19).

The new generation of "young" people who are added to the population have an additional choice to make, namely to join the work force immediately as unskilled labour $L$, or take some time to train themselves as skilled labour, or educate themselves to be scientists. This new generation tries to maximise the same type of objective function as in equation (55), subject to an income-budget constraint given by equation (17), and time-budget constraints that replace equation (56) with equations (13)-(15), but also retaining the evolution of $\tilde{H}_t$ as specified in equation (16). If they choose to invest in skilling or education, they need to work a sufficient length of time to be able to afford initial-period consumption.

This choice reflects the expected returns from each option, and is driven by the usual arbitrage conditions that are a subset of the first-order conditions of the individuals’ intertemporal utility-maximisation problem. The opportunity cost of the investment in building human capital as skilled labourers or scientists, is the wages, or leisure, or health-investment lost. The return from the investment is the higher wages earned from skilled labour or the profit earned from converting science education into profitable investment. These would be captured as variations to equation (60); they affect the supply of skilled and unskilled labour, and of scientists, from period $t + 1$ onwards.
4.1.2 Producers of the Final Product

The final-goods sector of the economy consists of a large number of competitive firms that use the production technology given by equation (25) to produce the final good \( Y \). These firms are price-takers in world markets. The price of the product they sell is governed by equation (54); in other words, the higher the "quality" of the product (reflecting the proportion of clean technology used in producing it), the higher the price-premium these producers receive in world markets.

Firms in this sector have to decide how much of each type of labour (skilled and unskilled), and of each type of capital or machine ("clean" and "dirty"), as well as the exhaustible resource \( \tilde{R} \), they use in producing the output.

They do this by solving the following profit-maximisation problem, taking the prices / costs of each input as given:

\[
\max_{\{L_{st}, L_{t}, m_{ct}, m_{dt}, \tilde{R}_t\}} \Pi_y^t = P_y^t Y_t - w_t L_{st} - w_t L_t - p_{mt} m_{ct} - p_{dt} m_{dt} - c(R) \times \tilde{R}_t; t = 0, 1, 2... \\
\]

where \( Y_t \) is given by equation (25); \( \Pi_y^t \) refers to the profit level from the production of the final product in period \( t \); \( P_y^t \) refers to the exogenous market price of the final product, determined in international markets; and, as a reminder, \( c(R_t) \) is the per unit extraction cost for the exhaustible resource, expressed in units of the final output (\( Y_t \)), where \( c \) is a non-increasing function of \( R \). Note that we have suppressed the index \( i \), as that additional level of detail does not add any new insights to the analysis that follows in this section; so we are working with average productivities \( (A_c, A_d) \) and a "typical" or "representative" machine within each sector ("clean" and "dirty").

Under suitable concavity assumptions for the profit function, the necessary and sufficient conditions characterising the solution of this problem state that, on the optimal (profit-maximising) time path, these firms hire or rent all inputs until the value of the marginal product of the input equals the price or rental price of the
where, in all cases for equations (65)-(69), $t = 0, 1, 2, \ldots$, and the five equations (first-order conditions) represent the demand functions for the five inputs into the production of the final good ($L_{st}, L_t, m_{ct}, m_{dt}, \tilde{R}_t$), respectively. In what follows, I use the notation:

$$y_{jt} = \left( \frac{Y_{jt}}{Y_t} \right)^{-\frac{1}{\gamma}}; j \in \{c, d\}$$

(70)

### 4.1.3 Machine Producers or Importers

The machines using both technologies are supplied (either manufactured or imported) by monopolistically competitive firms. These represent the intermediate-goods sectors. They produce (or import) the machines (the capital goods) that are sold to the final-goods sector. These firms gain their monopoly power by purchasing the design for a specific capital good from the research sector (the scientists). Owing to patent protection, only one firm manufactures (or imports) each machine.

Like the final-goods producers, these machine producers are also profit-maximisers. The profit-maximisation problem of the "representative" producer of machines, at time $t$ ($t = 0, 1, 2, \ldots$), working with technology $j \in \{c, d\}$, can be written as:

$$\max_{\{p_{m_{jt}}\}} \pi_{m_{jt}} = (p_{m_{jt}} - \psi) \times m_{jt}$$

(71)

subject to the demand functions for machines derived from equations (67) and (68):

$$m_{jt} = \left( \frac{\alpha y_{jt}}{p_{m_{jt}}} \right)^{\frac{1}{\gamma}} L_{jt} A_{jt}$$

(72)
It can be shown [see Acemoglu et al (2012) or Jones and Vollrath (2013)] that the profit-maximising price is a constant mark-up over marginal cost:

\[ p_{mj} = \frac{\psi}{\alpha} \]  

(73)

so that all machines sell at the same price. Substituting back into equation (72) yields the demand for the "representative" machine using technology \( j \in \{c, d\} \):

\[ m_{jt} = \left( \frac{\alpha^2 y_{jt}}{\psi} \right)^{\frac{1}{1-\alpha}} L_{jt} A_{jt} \]  

(74)

Thus the equilibrium profits of the "representative" machine producers of machine type \( j \in \{c, d\} \) can be written as follows:

\[ \pi_{m,j,t} = \psi \left( \frac{1}{\alpha} - 1 \right) \left( \frac{\alpha^2 y_{jt}}{\psi} \right)^{\frac{1}{1-\alpha}} L_{jt} A_{jt} \]  

(75)

Combining the profit-maximisation solution of the final-good producers with the profit-maximisation problem of the machine producers yields the following important result for our purposes, regarding relative equilibrium employment of the two types of labour working with the "clean" and "dirty" technologies [see Acemoglu et al (2012)]:

\[ \frac{L_{ct}}{L_t} = \left( \frac{c(R_t)^{\alpha_1} \alpha_2^2}{\nu^{\alpha_1 \alpha_2^2 (1+\alpha_2)^{\alpha_2}}} \right)^{(\theta_y - 1) - \alpha} \frac{A_{ct}^{\alpha - \nu}}{A_{dt}^{\alpha - \nu}} \]  

(76)

where \( \varphi_1 = (1 - \alpha_1)(1 - \theta_y) \). The higher the extraction cost \( c(R_t) \), the higher the amount of labour allocated to "clean" technology when \( \theta_y > 1 \).

### 4.1.4 Scientists, Technology and Productivity Growth

One of the possible outcomes of the typical individual's intertemporal utility maximisation problem is the investment of time that the individual makes in education, for acquiring skills towards becoming a scientist/researcher \( ED_t \).

As we explained in Section 3.7 above, the research sector, which is comprised of scientists, generates new ideas. Some of these ideas lead to productivity improvements in machines using the "dirty" technology, whereas others lead to further improvements in the use of "clean" technology.

In the stylised model, economic growth is driven by productivity growth. The evolution of productivity growth is defined by equation (28). It is clear from that
equation that the growth in the number of scientists, their success rate in innovating, and the productivity impact of that innovation are the key drivers of productivity growth. The growth in population (assisted by the growth of inward migration), the composition of migrants, and the decision of individuals on whether to pursue science as a career or not, are the key influences on the growth of scientists.

Knowledge provides the engine of growth, and the application of knowledge as between “clean” and “dirty” technologies determines the “quality” of growth. The allocation of scientists and engineers between the “dirty” and “clean” technologies, or equivalently the choice by a scientist or engineer as to whether they apply their skills towards improving the quality of the “dirty” or “clean” technologies or machines, reflects the relative expected profit of this choice.

To determine the profitability of research in each of the two areas, we need to start with the profitability of the machine producers for each of the two technologies, as given by equation (75). We then note that the profits of these machine producers are completely extracted by the scientists whose research leads to a new-design machine using each technology – i.e. to an actual invention. Since the probability of a successful innovation in technology $j \in \{c, d\}$ is $\eta_j \in (0, 1)$, and innovation increases the quality (or productivity) of a machine by a factor $(1 + \mu)$, it follows that the expected profit $\Pi_{jt}$ for a scientist engaged in research with technology $j$ at time $t$ is:

$$
\Pi_{jt} = \eta_j(1 + \mu)\psi\left(\frac{1}{\alpha} - 1\right)\left(\frac{\alpha^2y_{jt}}{\psi}\right)^{\frac{1}{\alpha}}L_{jt}A_{jt-1}
$$

It is the relative profitability of doing research with the two technologies that will drive where innovation is directed. This in turn will have a significant impact on the “quality” and sustainability of economic growth.

It can be shown [see Acemoglu et al (2012)] that, in equilibrium, the relative profitability of undertaking research in sector $c$ relative to sector $d$ is given by:

$$
\frac{\Pi_{ct}}{\Pi_{dt}} = \kappa, c(R_t)^{\alpha(\theta_y-1)}(1 + \mu\eta_cS_{ct})^{-\varphi-1}(A_{ct-1})^{-\varphi}\eta_d(1 + \mu\eta_dS_{dt})^{-\varphi(\alpha-1)}(A_{dt-1})^{-\varphi(\alpha-1)}\left(1 + \alpha_1\alpha_2\alpha_1^{2\alpha_1}\alpha_2^{2\alpha_2}\right)^{(\theta_y-1)}
$$

where, $\kappa = \frac{(1 - \alpha_1\alpha_2^{1+\alpha_2-\alpha_1}/(1-\alpha_1)}{\alpha_1^{2\alpha_1}\alpha_2^{2\alpha_2}}$.

What equation (78) says is that, so long as $c(R_t)$ is decreasing in $R_t$, and the two technologies are gross substitutes (i.e. $\theta_y > 1$), as the exhaustible resource gets depleted the incentives to direct innovations towards the clean technology increases. So the market mechanism is working in the stylised model; whether it
needs support to work more effectively and efficiently is a matter we address later in Section 5.3 below.

### 4.2 Market-Clearing Conditions

There are six market-clearing conditions - for the final product, unskilled labour, skilled labour, scientists, machines and international exchange of goods and machines.

#### 4.2.1 Final Good

Market clearing for the final good implies, for \( t = 0, 1, 2, \ldots \):

\[
C^y_t + Z_t \chi = Y_t - X^y_t - \psi \left( \int_0^1 m_{ci} dt + \int_0^1 m_{di} dt \right) - c(R_t) \tilde{R}_t - \tau Y_t
\]

where, all costs and taxes are measured in units of the final output, \( C^y_t \) is aggregate domestic consumer demand for the final product, \( Z_t \chi \) represents the wedge between output and consumption arising from the potential distortionary effects of government activities, \( X^y_t \) is the export of the final product, \( \psi \) is the unit cost of producing (or importing) any machine (and the term in brackets represents the total quantity of machines used in production in period \( t \), i.e. \( M_t \)), the fourth term on the right hand side measures the total cost of the exhaustible resources used in producing the final output in period \( t \), and the final term represents the total tax collected in period \( t \) to pay for all the services provided by the government.

#### 4.2.2 Labour

Market clearing for unskilled labour and skilled labour can be represented as follows:

\[
\begin{align*}
L_t^d &= L_t^s \\
L_{st}^d &= L_{st}^s
\end{align*}
\]

where the superscripts \( d \) and \( s \) refer to demand and supply respectively.

The demand for unskilled labour and skilled labour for \( t = 0, 1, 2, \ldots \) are derived

\[
Z_t \equiv c_{\Omega_w}(\Omega_w) + c_{\Omega_y}(\Omega_y) + c_{B}(B)
\]

as in equation (48) [see Acemoglu et al (2014)].
from the profit-maximisation conditions of the final-output producers, as presented in Section 5.1.2.

The supply of both types of labour for \( t = 0, 1, 2, \ldots \) has two sources: the solution of the optimisation problem of the consumers in Section 5.1.1, and the inflow of labour from overseas:

\[
L_t^s = L_t + (F_t^L - F_{t-1}^L) \\
L_{st}^s = L_{st} + (F_t^{Ls} - F_{t-1}^{Ls})
\]  

(82)

(83)

4.2.3 Scientists

Market clearing for scientists takes the following form:

\[
Sc_{ct} + Sc_{dt} = Sc_t
\]

(84)

where \( Sc_t \) refers to the total number of scientists available at the beginning of period \( t \), who choose to target working with the "clean" \( (Sc_{ct}) \) or "dirty" \( (Sc_{dt}) \) technologies for the period ahead.

The supply of both types of scientists for \( t = 0, 1, 2, \ldots \) has two sources: the solution of the optimisation problem of the consumers in Section 5.1.1, and the inflow of scientists from overseas:

\[
Sc_t^s = Sc_t + (F_t^{Sc} - F_{t-1}^{Sc})
\]

(85)

4.2.4 Machines

Market clearing for machines in aggregate takes the following form:

\[
M_{jt}^d = M_{jt}^s
\]

(86)

for \( j \in \{c, d\} \), where the aggregate domestic demands for the two classes of machines (the left-hand side of the inequalities), for \( t = 0, 1, 2, \ldots \), are derived from the profit-maximisation conditions for final-output producers, as presented in Section 5.1.2.

The supply of new machines to each sector reflects the successful innovations introduced by scientists, which in turn determines the total domestic production and/or imports of machines which embody these innovations.
4.2.5 Exhaustible Natural Resources

In the version of the model we are working with, there is no private property rights on the exhaustible natural resource, so it is inappropriate to refer to "market clearing conditions".

An alternative condition is that the demand for the resource in production is less than the available stock:

\[ \tilde{R}_t \leq R_t \]  

(87)

where the left-hand side of the inequality (the demand for the natural resource), for \( t = 0, 1, 2, ... \), is derived from the profit-maximisation conditions for the final-goods producers, as presented in Section 5.1.2.

The stock of the natural resource (the right-hand side of the inequality) is given by the equation of motion (26).

One of the possible policy levers available to the government is to assign private property rights to natural resources; this is an option we return to in Section 5.3.1.

4.2.6 International Payments Balance

The stylised small open economy can export the single final (consumable) product and can import "clean" machines.\(^{33}\) In the short run, it is a price-taker for both products in international markets. However, in the longer run, by increasing the weight of the use of "clean" technology in the production of the final good, it can increase the premium it earns on the production and sale of the final good.

Without loss of generality, we assume that the exchange rate is fixed at a value of one, and specify the condition for the international payments balance of this small open economy as follows:

\[ P^y_t \times (Y_t - C^y_t - S\alpha_t) = p^m_{m \ell_t} \times (F^{M_{\ell_t}}_{t+1} - F^{M_{\ell_t}}_t) \]  

(88)

where, \( X^y_t = (Y_t - C^y_t - S\alpha_t) \). Thus, the relative price of the final product \( (P^y_t) \) viz the price of the "clean" machine \( (p^m_{m \ell_t}) \) is also the small open economy's terms of trade.

\(^{33}\) Given the purpose of this paper, complicating the model by adding final products that can also be imported, or switching to importing the final product but exporting the machines does not add anything of value to our analysis.
4.3 Equilibrium

We analyse and compare the long-run (steady state) equilibrium properties of three versions of the model. The first version, which serves as a very useful benchmark, is a closed-economy model with no government intervention towards influencing economic, environmental and/or social outcomes, where the only externalities present relate to natural resources and the environment. This is essentially the laissez-faire version of the model presented and analysed by Acemoglu et al (2012). The second is the LSF model presented in this paper, but in the absence of deliberate public policy (i.e. the laissez-faire version of the LSF model). The final version is the full model presented in this paper, with an explicit role for public policy towards enhancing intergenerational wellbeing. Some of the key equilibrium outcomes of these three models are presented in the Table at the end of this section, where "LF" refers to "Laissez Faire".

4.3.1 Laissez-Faire - A Benchmark Model

The model here is essentially the closed-economy version of the model used in this paper, excluding references to social cohesion, equity, resilience or the Chichilnisky extension; and where utility is both a function of the consumption of the single output ($Y$) and the quality of the environment (leisure and good health are excluded), with both directly entering the individual’s utility function. Thus on the consumption side, the equivalents of equations (9)–(12) hold (with the exclusions just referred to) and the production side of the economy is identical to the one used here, except for no differentiation between “skilled” and “unskilled” labour – labour is perfectly substitutable between “clean” and “dirty” technologies – and where “good health” affects neither the individuals’ utility nor the productivity of labour.

Such an equilibrium is given by sequences of: final-good output ($Y_t$), unskilled wages ($w_t$), prices for machines ($p_{jit}$), demands for machines ($m_{jit}$), labour demands ($L_{jd}^t$) for use with the two technologies $j \in \{c, d\}$, allocations of scientists (or research) to “dirty” and “clean” technologies ($S_{cdt}$, $S_{cd}$) respectively, use of the non-renewable natural resource ($\tilde{R}_t$), and the quality of the environment ($E_t$), such that in each period $t = 0, 1, 2, \ldots$:

- ($Y_t$, $\tilde{R}_t$) maximise the profits of the final-good producers;
- ($p_{jit}$, $m_{jit}$) maximise profits for the producers of machines in sector $j$, $j \in \{c, d\}$;
• \((S_{cd}, S_{c_d})\) maximise the expected profits of researchers choosing to work with “dirty” and “clean” technologies respectively;

• \((w_t)\) clears the labour market;

• the final-goods market clears as per equation (79) – except for \(r = 0\); and

• the evolution of the exhaustible resource is given by (26), and that of the quality of the environment by (38).

As Acemoglu et al (2012) demonstrate, both the dynamics and the equilibrium properties of this model critically depend on the degree of substitution between the two technologies (“clean” and “dirty”) in the production of the final output.

The exhaustible resource is a key ingredient in the use of the “dirty” technology. Going back to equations (78) which show the relative expected profitability from research with the two technologies, and assuming \(\theta_y > 1\) and \(c(R_t)\) is decreasing in \(R_t\), as the non-renewable (exhaustible) resource stock gets depleted, the market generates the incentives for research to direct innovation towards the “clean” technology, and leads to a rise in the demand for \(L_c\) relative to \(L\) [see equation (76)].

Acemoglu et al (2012) prove that, under these circumstances, innovation will eventually be directed to the clean technology only, and the long-run equilibrium growth rate of the economy will be \(\mu \eta_c\). Provided the initial quality of the environment and the value of \(\theta_y\) are sufficiently high, the switch to clean technology occurs faster and an environmental disaster is avoided. We thereby achieve a positive and sustainable economic growth rate, with a positive sustainable level of wellbeing.

We summarise the equilibrium outcomes of this benchmark laissez-faire model in the first column of the Table at the end of this Section. It is important to note that this model does have a sustainable positive equilibrium growth rate with a positive level for intergenerational wellbeing - but only because wellbeing is a function of a limited set of consumable goods and services (and few externalities are allowed). And in any case, even under its own terms, this equilibrium is not socially optimal.

The fundamental rationales for the latter conclusion (i.e. the sub-optimality of the laissez-faire equilibrium) are four-fold [see Jones and Vollrath (2013), Chapter 5; or Acemoglu et al (2012)], all related to externalities or market structures: first, scientists are not able to internalise the positive externalities associated with their research (researchers are not compensated for their contribution towards improving the productivity of future researchers, so there is too little research); second,
the negative environmental externality created by the use of "dirty" technology is not fully internalised; third, the private cost of extraction $c(R_t)$ does not reflect the total social (scarcity) value of the exhaustible resource; and finally, there is the standard (static) monopoly distortion (the "consumer-surplus effect") – an inventor of a new design (for a machine) captures the monopoly profit but not the entire potential gain to society of the invention, which results in too little innovation.

In this version of the model, the government, through appropriate policies, can incentivise private actors to move the economy towards the "socially optimal" solution. Acemoglu et al (2012) show that this can be achieved through the combination of the following policies: a "carbon tax" (i.e. a tax on the use of the "dirty" technology, or tasks); a Research and Development (R&D) subsidy for "clean" research; a subsidy on the use of all machines (using "dirty" or "clean" technology); and a permanent resource tax (all proceeds from taxes/subsidies being redistributed/financed lump sum). The intended collective impact of this package of policies is to switch production towards the use of clean technology, with a view to increasing the long-term sustainable growth rate of the economy.34

By way of demonstration, a subsidy for "clean" research, represented through an adjustment to equation (77), would take the following form:

$$\Pi_{ct} = (1 + q_t) \eta_c(1 + \mu) \psi \left( \frac{1}{\alpha} - 1 \right) \left( \frac{\alpha y_{ct}}{\psi} \right)^{\frac{\gamma}{1 - \alpha}} L_{ct} A_{ct-1} \quad (89)$$

where $q_t$ represents the subsidy rate.

It is worth highlighting that the subsidy in support of the use of machines is to correct for the under-utilisation of machines due to monopoly pricing in the laissez-faire equilibrium. In addition, the user-cost of the exhaustible resource is determined by the cost of extraction and does not reflect its scarcity value.

Regarding this last point, an alternative policy option open to the government, which is again intended to provide the maximum opportunity and incentives for the private sector to be guided towards a "socially optimal" outcome by changing the structure of production, would be to establish well-defined private property rights to the exhaustible resource, and vest these in price-taking infinitely-lived profit-maximising firms [see Acemoglu et al (2012), which also has references to related earlier literature].

An important result highlighted in the Acemoglu et al (2012) paper relates to whether government policy intervention needs to be temporary or permanent in nature, to incentivise the switch of production towards the use of clean technology, thus generating socially optimal outcomes. They show that a critical influence on this choice is the degree of substitutability between the clean and dirty technologies in the production of the final output.
To gain insight into the optimal solution in this case, note that the fundamental intuition behind dynamic (or intertemporal) optimisation is that optimality requires that resources (and consumption) be allocated over time in such a way that there are no favourable (i.e. wellbeing-increasing) opportunities left for intertemporal re-arrangements of production or consumption. Applying this broad principle in the context of the problem at hand, as specified in the previous paragraph, Hotelling (1931) showed that, in a competitive industry with a privately-owned exhaustible resource, the price \( p \) of the exhaustible resource must change over time so that net rents increase at the rate of interest \( r \), i.e.

\[
\frac{p_{t+1} - c}{p_t - c} = 1 + r; t = 0, 1, 2, ...
\]  

(90)

where \( c \) is the constant marginal cost of extracting one unit of the resource, and \( r \) is the rate of interest – a result known as Hotelling’s Rule. Otherwise, there would be opportunities for profitable arbitrage.

In our context, and using the following utility function just to demonstrate the point simply:

\[
u(C_t, E_t) = C_t^{1-\sigma} + v(E_t)
\]  

(91)

where \( \sigma \) is a constant coefficient of relative risk aversion in consumption, \( \partial v / \partial E > 0 \) and \( \partial^2 v / \partial E^2 < 0 \), Acemoglu et al (2012) show that the Hotelling Rule leads to the following result:

\[
r = (1 + \rho) (1 + g)^\sigma - 1
\]  

(92)

where \( g \) is the asymptotic growth rate of consumption.

The reason for labouring this point at this juncture is that it brings the discount rate \( \rho \) into the picture and, as Acemoglu et al (2012) show, highlights the circumstances under which laissez-faire equilibrium could be a “disaster” for the environment, and therefore utility. To be more specific, if both the elasticity of substitution between “dirty” and “clean” technology in production and the discount rate are jointly sufficiently low, then a “disaster” cannot be avoided under laissez-faire – and establishing private property rights on exhaustible resources may not provide the full solution towards a sustainable equilibrium, on its own.

4.3.2 Laissez-Faire - LSF Model

We now consider the laissez-faire equilibrium properties of our expanded small-open-economy model where, in addition to exhaustible resources and the environment, we also include externalities associated with social cohesion, resilience to
systemic risks and equity; and the Chichilnisky extension is included but then suppressed (see below), because there is no role for government in the laissez-faire model. The individuals’ utility functions now include, in addition to the consumption of the single output \((Y)\), also the consumption of time (or "leisure" in its most general sense) and "good health". Finally, we now conceptualise "skilled" labour and scientists as "human capital", which requires investment to convert investment-time into skilled labour or scientists.

Such an equilibrium is given by sequences of: final-good output \((Y_t)\); final-good price \((P^y_t)\); consumption of products, time, and good health respectively \((C_y,t, C_x,t, C_h,t)\); final-good export \((X^y_t)\); unskilled wages \((w_t)\) and skilled wages \((w_{st})\); prices for machines \((p_{jt})\) and demands for machines \((m_{jt})\), \(j \in \{c,d\}\); machine productions \((m_{jt})\) using technologies \(j \in \{c,d\}\); stock of foreign "clean" machines imported \((F^{M_c}_t)\); qualities of machines \((A_{jt})\) using technologies \(j \in \{c,d\}\); skilled-labour demand \((L^{sd}_t)\) and unskilled-labour demand \((L^{st}_t)\); human capital formation through skilling (i.e. supply of skilled labour) \((L^{sd}_s,t)\) or education (i.e. supply of scientists) \((L^{ss}_s,t)\); allocations of scientists (or research) to "dirty" and "clean" technologies \((S_{c,d,t}, S_{c,c,t})\) respectively; stocks of foreign skilled and unskilled labour and scientists \((F^{L_s}_t, F^{L_s}_s, F^{Sc}_t)\); use of the non-renewable natural resource \((\tilde{R}_t)\); quality of the environment \((E_t)\); social capital \((K_{s,t})\); degree of social cohesion \((S_t)\); degree of resilience to potential systemic risks to economic and social infrastructure and the environment \((\Gamma_{y,t}, \Gamma_{S,t}, \Gamma_{E,t})\) respectively; degree of inequity \((Q_t)\) and degree of poverty \((POV_t)\); and economic infrastructure \((\Lambda_{y,t})\); such that, in each period \(t\):

- \((Y_t, \tilde{R}_t)\) maximise the profits of the final-good producers;
- \((p_{jt}, m_{jt})\) maximise profits by the producers of machines using each technology \(j, j \in \{c,d\}\);
- \((S_{c,d,t}, S_{c,c,t})\) maximise the expected profits of researchers choosing to work with "dirty" and "clean" technologies respectively;
- \((w_t, w_{st})\) clear the unskilled and skilled labour markets respectively;
- \((X^y_t, P^y_t, F^{M_c}_t)\) clear the international good and capital balance;
- all other market-clearing conditions set out in Section 5.2 hold; and
- the evolutions of: the exhaustible resource is given by (26); the quality of the environment by (38); social cohesion by (39); inequity by (34); poverty by (41); economic infrastructure by (44); economic resilience by (45); social capital by (46); social and environmental resilience by (40); the quality (or productivity) of the machines by (28); inward migration of people and machines by (50)-(53); unskilled labour by (82); skilled labour by (14) and (83); scientists by
(15) and (85); physical capital (machines) by (22) and (88); and the quality of health by (16).

The absence of government interjection in the laissez-faire scenario is captured by setting $\tau = 0$ in equation (79), and $\delta_k = 0$, $k \in \{E, \Lambda_y, \Gamma_y, S, POV, \Gamma_E, \Gamma_S, K_s\}$, in equations (38, 39, 40, 41, 44, 45, 46) respectively; as well as setting $\omega = 1$ in equation (47) [this automatically eliminates the Chichilnisky extension].

The differences between the two laissez-faire versions of the model, presented in the previous section and the current section, arise because of the way the wellbeing function is specified, including additional public goods or externalities through which wellbeing are affected: social cohesion, equity, and resilience to systemic risks.

The fundamental economic-dynamic properties of the laissez-faire (1) model of the previous section are not affected by the extensions introduced in this section, in the specific sense that economic growth is still driven by the growth and composition of technology, in turn reflecting where innovations occur, as well as the composition of the labour force. The introduction of inward migration, and possible imports of machines and exports of the single final good, do not affect this basic outcome, although they do add other avenues through which the evolution of the size and composition of the labour force, scientists and machines are affected.

We now care about both the rate and the composition of economic growth, not only because this (may) affect the potential sustainable growth rate of the economy, but also because it affects equity, which in turn has an effect on social cohesion. Another channel that is opened in this model, inward migration, has an effect on both potential economic growth and equity, as well as on social cohesion.

These additional externalities open up further channels through which the expanded laissez-faire equilibrium of this section may not be socially optimal. It is not only the externalities associated with environmental influences that may not be fully internalised, but also those associated with the additional spheres of wellbeing the LSF model introduces.

The market- and/or community-driven mechanisms may not be sufficient to fully internalise the potential negative externalities caused by environmental or economic-infrastructure degradation caused by production [equations (38) and (44)], and/or the potential negative impact on social cohesion of inward migration, or rising

---

35 Whether we set these policy-parameter values to zero or to an arbitrary minimum positive value does not affect the qualitative results presented here. $\xi_j$ and $\gamma_j$, for all $j$, are treated as fixed parameters for convenience.
inequity, or rising poverty [equation (39)]. Based on the specification of the individual utility and aggregate wellbeing functions [equations (9)-(12), (35)-(36), and (47)-(48)], this leads to a potential failure to achieve the full wellbeing-potential of the economy and society.

We summarise the equilibrium outcomes of the laissez-faire scenario for this model in the second column of the Table at the end of this Section.

In the absence of deliberate regeneration policies directed to economic infrastructure, the environment and social cohesion, ongoing economic growth, decreasing equity and deteriorating social cohesion will push overall wellbeing to a level that is below potential. This is not to deny that, even in the absence of government intervention (i.e. deliberate collective action funded through a tax on production), faced by pressures on social cohesion, economic and social resilience etc, the community would find ways of gathering resources and investing towards addressing these issues. However, these may be inadequate because the benefits of such actions may not be fully internalised. Thus the scope and breadth of possible economic, environmental and social policies need to be expanded beyond environmental and economic growth considerations to help us get towards a socially optimum solution. In addition, in this expanded model we can also enhance overall wellbeing by improving economic and social resilience to the systemic risks that our economic and social infrastructure are exposed to.

4.4 Equilibrium under Wellbeing-Maximisation Policy

Now consider the full model presented in this paper. The socially optimal (wellbeing maximisation) solution of this model is a dynamic path for: final-good output \( (Y_t) \); final-good price \( (P^y_t) \); consumption of products, time, and good health respectively \( (C_y,t, C_x,t, C_h,t) \); final-good export \( (X^y_t) \); unskilled wages \( (w_t) \) and skilled wages \( (w^s_t) \); prices for machines \( (p_{jt}) \) and demands for machines \( (m_{jt}) \), \( j \in \{c, d\} \); machine productions \( (m_{jt}) \) using technologies \( j \in \{c, d\} \); stock of foreign "clean" machines imported \( (F^M_{jt}) \); qualities of machines \( (A_{jt}) \) using technologies \( j \in \{c, d\} \); skilled-labour demand \( (L^{d,s}_{dt}) \) and unskilled-labour demand \( (L^s_{st}) \); human capital formation through skilling (i.e. supply of skilled labour) \( (L^{s,s}_{st}) \) or education (i.e. supply of scientists) \( (Sc^s_{st}) \); allocations of scientists (or research) to "dirty" and "clean" technologies \( (Sc^d_{dt}, Sc^c_{ct}) \) respectively; stocks of foreign skilled and unskilled labour and scientists \( (F^L_{it}, F^{L,s}_{it}, F^{Sc}_{it}) \); use of the non-renewable natural resource \( (\bar{R}_t) \); quality of the environment \( (E_t) \); social capital \( (K_{st}) \); degree of social cohesion \( (S_t) \);
degree of resilience against potential systemic risks to economic and social infrastructure and the environment \((\Gamma_{y,t}, \Gamma_{S,t}, \Gamma_{E,t})\) respectively; degree of inequity \((Q_t)\) and degree of poverty \((POV_t)\); and economic infrastructure \((\Lambda_{y,t})\) - that maximises the intertemporal wellbeing function \((47)\), subject to the:

- utility function \((36)\);
- production function \((42)\);
- the evolutions of: the exhaustible resource \((26)\), the quality of the environment by \((38)\), social cohesion \((39)\), inequity \((34)\), poverty \((41)\), economic infrastructure \((44)\), economic resilience \((45)\), social capital \((46)\), social and environmental resilience \((40)\), the quality (or productivity) of the machines \((28)\), inward migration of people and machines \((50)-(53)\), unskilled labour \((82)\), skilled labour \([(14)\ and \ (83)\] , scientists \([(15)\ and \ (85)\] , physical capital (machines) \([(22)\ and \ (88)\] , and the quality of health \((16)\);
- market clearings for the final good \((79)\) , scientists \((84)\) , unskilled labour \((80)\) , skilled labour \((81)\) ;
- international payments balance \((88)\) ; and
- the government budget constraint \((49)\).

We summarise the qualitative (comparative) equilibrium outcomes of the wellbeing maximisation problem in the last column of the Table below.

As we stated before, under the LF(1) scenario (column 1), innovation will eventually be directed to the clean technology only, and the long-run equilibrium growth rate of the economy will be \(\mu\eta_c\), under suitable assumptions about the degree of substitutability between “dirty” and “clean” technology in the production of the final output. Provided the initial quality of the environment and the value of \(\theta_y\) are sufficiently high, the switch to clean technology occurs faster and an environmental disaster is avoided. Thus we achieve a positive and sustainable economic growth rate, with a positive sustainable level of wellbeing.

However, in the context of this paper, this result is misleading because it does not allow for the full set of externalities included in the LF version of the LSF model [LF(2) in column 2]. Once these additional externalities are allowed for, there is no longer a guarantee that the LF(2) model will converge to a sustainable equilibrium with a positive output-growth rate and a positive level of wellbeing, even under the favourable assumptions of the LF(1) model.
Table 1: Summary of Equilibrium Properties of Alternative Models

<table>
<thead>
<tr>
<th></th>
<th>LF (1)</th>
<th>LF (2)</th>
<th>W-Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>W</td>
<td>&gt; 0</td>
<td>→ 0</td>
</tr>
<tr>
<td>2</td>
<td>( \dot{g}_y )</td>
<td>( \mu \eta_c )</td>
<td>→ 0</td>
</tr>
<tr>
<td>3</td>
<td>( \dot{\omega} )</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>E</td>
<td>∈ (0, 1)</td>
<td>→ 0</td>
</tr>
<tr>
<td>5</td>
<td>Q</td>
<td>-</td>
<td>( \max )</td>
</tr>
<tr>
<td>6</td>
<td>S</td>
<td>-</td>
<td>→ 0</td>
</tr>
<tr>
<td>7</td>
<td>( \Gamma )</td>
<td>-</td>
<td>→ 0</td>
</tr>
</tbody>
</table>

In any case, for reasons similar to the ones given for the LF(1) model, even if a sustainable equilibrium with a positive output-growth rate and positive sustainable wellbeing exists, it is most unlikely to be socially optimal for similar reasons to those given for the sub-optimality of the LF(1) equilibrium outcomes. What column 3 shows is the comparative outcomes of the LSF model when policies are put in place that can effectively and efficiently deal with the externalities that have been identified in model LF(2).

The key results can be summarised as follows.

First, the equilibrium growth rate of the wellbeing-optimisation problem, should a solution exist, will be less than \( \mu \eta_c \) [row 2 in the Table]. To see this all we need to do is to observe, by way of demonstration, that to the extent that the implications of that positive growth rate on economic infrastructure [equation (44)], the implications of the structure of production and of employment on equity [equations (29)-(34)], and the implications of the evolution of equity itself on social cohesion [equation (39)] are excluded, and no policies are in place to correct for their social effects, both externality functions and therefore the wellbeing function will tend towards zero (or to a level below potential) [row 1].

Second, whether such a sustainable wellbeing-maximising solution exists, and if it does what the level of the associated sustainable economic growth rate will be, will depend on whether the government can effectively and efficiently provide the externalities summarised by the functions \( \Omega_w(E_t, S_t, \Gamma_{ES,t}) \) and \( \Omega_y(\Lambda_{y,t}, \Gamma_{y,t}, K_{s,t}) \) in equations (35) and (42), respectively. These solutions will also depend on how much weight we put on the wellbeing of generations in the very distant future [the value of parameter \( \omega \) in equation (47)] [row 3].

Third, should we be able to switch production effectively and efficiently to the use of “clean” technology [row 4], and also provide the appropriate economic and social infrastructure, we will be significantly increasing our chances of en-
joying positive sustainable wellbeing accompanied and supported by a positive sustainable economic growth rate, high degree of social cohesion (supported by low inequity and low poverty) [rows 5, 6], a clean environment and good health [row 4], and enhanced resilience to major systemic shocks [row 7]. That would indeed be a New Zealand where talent would love to live. Column 3 [row 1] in the Table captures the outcomes of such a complementary collection of effectively and efficiently implemented wellbeing-maximising policies.

The critical additional tradeoffs that the optimisation conditions of the expanded wellbeing-maximisation problem introduce for consideration in formulating public policy include: the positive and negative impacts of inward migration on skills and technological progress, versus social cohesion respectively; the direct and indirect wellbeing costs versus benefits, of investments in enhancing environmental, economic and social resilience to potential major shocks; the direct and indirect impacts of the change of the structure of production and employment on equity, and through that on social cohesion; the impact on overall wellbeing of the weight we put on the wellbeing of generations way into the future; and the costs of taxation, versus the benefits of the externalities generated by the use of the funds collected through taxation.

An appropriate policy package that offers a high chance of providing or enhancing the externalities that are wrapped around comprehensive wealth in the Living Standards Framework, while also effectively and efficiently dealing with the complementarities and tradeoffs highlighted in the preceding paragraph, would augment Acemoglu et al’s (2012) policy package (targetted at enhancing the LF(1) model, as presented above), with the following - all of which operate through the equations of motion for externalities. The policy instruments are direct investments, subsidies and taxes.

- **Potential economic growth** can be increased through investments in economic infrastructure (which is a component of our aggregate economic capital), as well as in improving economic resilience; subsidies for skilling, education and health-improvements, towards increasing the domestic supply of healthy skilled labour and scientists; and controls and subsidies towards encouraging the immigration of skilled labour and scientists. In the stylised model, the main direct policy levers through which policy instruments would affect overall wellbeing are $\delta_{s,y}$ and $\delta_{r,y}$ in equations (44) and (45); $\mu_{j}, j \in \{tr, ed, h\}$ in equations (14)-(16); and possible direct subsidies introduced into equations (51) and (52).

- **Sustainability** of comprehensive wealth can be enhanced by influencing the
structure of production (favouring "clean" technology), through appropriate R&D, skilling, education and immigration subsidies, a "carbon tax" (i.e. a tax on the use of the "dirty" technology), appropriate pricing of natural resource use, and direct or joint (private-public) investment in environmental regeneration. Direct policy levers are $\delta_E$ in equation (38); the same levers on training, education and immigration as those referred to under sustainability above; and subsidies for "clean research" and taxes on "dirty" technology that can be introduced into equation (77) [as in equation (89), by way of example].

- The evolution of equity, which is driven by equation (34) in the stylised model, can be influenced by incentivising more investment in training and education (as above), to be able to produce more skilled labour, and scientists and engineers; and in general investing towards incentivising people to seek and find productive work (i.e. through "social investment", to use the language of the current policy framework in New Zealand) [this could be effected by influencing the value of the social capital coefficient $(\delta_{K_s})$ in equation 39]. There is also a second sense in which we use the term equity in our model; this relates to intergenerational equity as captured by the Chichilnisky extension – see equation (47); the main policy lever here is the value of the parameter $\omega$ which the policy maker can set to a positive value.

- Social cohesion can be enhanced by qualitative inward migration controls (as above), investing in the teaching of different languages and cultures, actively encouraging the mixing of communities, as well as equity-improving and poverty-reducing measures (i.e. investments in social capital). Social cohesion in our model evolves according to equation (39). Policy levers are $(\delta_{K_s})$ in equations (39) and 46); and $(\delta_{POV})$ in equation (41).

- Building resilience is about enhancing economic, environmental and social resilience to potential systemic shocks. The relevant equations of motion are equations (40) and (45). Policy levers are $\delta_{\Gamma_S}$, $\delta_{\Gamma_E}$ and $\delta_{\Gamma_y}$.

The common thread across all the ingredients of this whole complementary [to Acemoglu et al (2012)] policy package is that they all positively contribute to individual and communal wellbeing by influencing the growth, distribution and protection of the capital assets that comprise comprehensive wealth.
5 Concluding Comments and Next Steps

The Living Standards Framework (LSF) is intended to provide a guide for thinking about good economic, environmental and social policies in an integrated way. The main burden of this paper was to demonstrate that a unified stylised model can be constructed to help us do that. We have made some progress towards that end by presenting the skeleton of such a model. To re-emphasise a point we made at the very beginning, our suggested model is one (just one) of possibly several alternatives for achieving the same purpose.

The question has been asked, what have we learned as a result of this effort that we did not know before?

Once we specify the generation of shared (across society and generations) and sustainable wellbeing as the main purpose of public policy, and we appreciate the multiplicity and complementarity of spheres of wellbeing, it becomes self-evident that we need to think of economic, social and environmental policies in an integrated way. The LSF provides a framework to facilitate such integrated thinking and policy formulation. Ignoring these linkages in policy formulation will potentially lead to policies that harm intergenerational individual and communal wellbeing. The stylised model we have suggested highlights the key complementarities and tradeoffs between policies, as well as their outcomes, as we pursue higher sustainable and shared wellbeing.

Policy making is particularly exciting in the pursuit of a shared vision for the future of New Zealand. It is in this context that the focus shifts strongly to a search for mutually reinforcing actions that expand our collective wellbeing-frontier; tradeoffs are replaced with complementarities as the main focus of policy advice. And once that vision is understood and shared, democracy endogenises and builds support for wellbeing-enhancing and time-consistent decisions.

Policy formulation in our stylised model has been presented in the context of an intertemporal optimisation of a social wellbeing function, subject to a series of constraints. That exposes us to the fair criticism that our model implicitly conceptualises the government as "a machine that computes optimal solutions to social welfare maximands" [Romer (1988), p. 167], or a social engineer or despot that pursues an "externally defined, supra-individualistic ideal" [Buchanan (1986), p. 5]. That is not our conceptualisation of the government or its role; the
intertemporal social-wellbeing optimisation setting has been used for convenience only.

We conceptualise governments as collective agents through whose activities public goods are provided or their provision is facilitated, and the associated externalities are realised. The purpose of all such collective action is to enhance (note, enhance or improve - not equalise) the opportunities and capabilities of individuals to improve their private wellbeings. The government gives effect to its intent by incentivising and supporting private individuals, businesses and communities to behave in ways that align with overall wellbeing. The democratic process provides the machinery for the ongoing alignment of government action with the pursuit of private interests (i.e. the evolution of government action is endogenised through the democratic process, supported by appropriate institutions).

Our next step in modelling is to capture this intent by using viability theory rather than optimisation techniques in representing the behaviour of the government. Under this approach, the focus of the policy maker shifts to ensuring viable boundaries within which private utility- and profit-maximising activities can be pursued in a sustainable manner. Hopefully, this will remove any concerns about the representation of the government as a social engineer or despot that pursues an "externally defined, supra-individualistic ideal". This alternative technique may also make it possible to parameterise the model for use in policy simulations [Krawczyk and Judd (2015), Krawczyk and Kim (2014)].

Appendix: Dynamic Optimisation

In this section we sketch out the mathematical method we have used throughout this paper to explore the equilibrium properties of the various models.

Generically speaking, we are working with various versions of an infinite-horizon constrained dynamic optimisation problem. The basic structure of these problems takes the following shape:

\[
\max_{a_t \in A_t} \sum_{t=0}^{\infty} \beta^t f_t(a_t, s_t) \\
\text{subject to } s_{t+1} = g_t(a_t, s_t), \ t = 1, 2, 3, ... \\
given \ s_0.
\]

To ensure that the total discounted sum is finite, it is assumed that \( f_t \)
is bounded for every $t$ and $\beta < 1$; $a_t$ is known as a control variable (it is under the control of the decision-maker and any value $a_t \in A_t$ may be chosen); and $s_t$ is referred to as the state variable and it is determined indirectly through the transition equation $s_{t+1} = g_t(a_t, s_t)$, which describes the transition of one state ($s_t$) into the next state ($s_{t+1}$). Although both the control and state variables can be vectors, here we assume that they are scalars. Various additional constraints may be imposed on the state variables. In particular, in economic models, negative values may be infeasible.

Given the initial state $s_0$, suppose that $A_t$ is open for every $t$, and that $f_t$ and $g_t$ are concave and increasing in $s_t$; then $a_t$ ($t = 0, 1, 2, \ldots$) is an optimal solution if and only if there exist unique multipliers $\lambda_t$ ($t = 0, 1, 2, \ldots$) such that:

$$D_{a_t} f_t(a_t, s_t) + \beta \lambda_{t+1} D_{a_t} g_t(a_t, s_t) = 0 \quad t = 0, 1, 2, \ldots$$

$$D_{s_t} f_t(a_t, s_t) + \beta \lambda_{t+1} D_{s_t} g_t(a_t, s_t) = \lambda_t \quad t = 0, 1, 2, \ldots$$

$$s_{t+1} = g_t(a_t, s_t) \quad t = 0, 1, 2, \ldots$$

where $D$ is the partial derivative operator $[D_{a_t} f_t(a_t, s_t) \equiv \partial f_t(a_t, s_t)/\partial a_t]$.

Where the problem is stationary (i.e. $f$ and $g$ are independent of $t$), it may be reasonable to assume that the optimal solution converges to a steady state in which variables are constant, that is:

$$a_t = a^*$$

$$s_t = s^*$$

$$\lambda_t = \lambda^*$$

$$D_{a^*} f(a^*, s^*) + \beta \lambda^* D_{a^*} g(a^*, s^*) = 0$$

$$D_{s^*} f(a^*, s^*) + \beta \lambda^* D_{s^*} g(a^*, s^*) = \lambda^*$$

$$s^* = g(a^*, s^*)$$

These conditions can then be used to analyse the properties of the steady state.

The fundamental intuition behind dynamic (or intertemporal) optimisation is that optimality requires that resources (and consumption) be allocated over time in such a way that there are no favourable opportunities left for intertemporal re-arrangements of production or consumption towards further increasing the value of the objective function.
References


Au, Joey; Coleman, Andrew; Sullivan, Trudy (2015). "When I’m 64 – what do New Zealanders want from their retirement income policies?", New Zealand Treasury Note.


Gleisner, Ben; McAlister, Fiona; Galt, Margaret; Beaglehole, Joe (2012). "A Living Standards Approach to Public Policy Making," New Zealand Economic Papers; Vol. 46, pp. 211-238.


New Zealand Ministry of Social Development (NZMSD) (various years). *Social Report*.


